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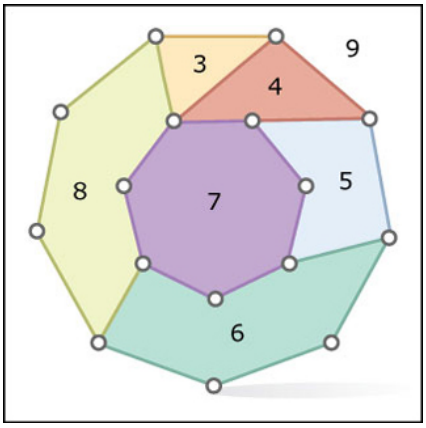
Chapter 01: Combinatorial analysis



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Chapter 1

combinatorial analysis

1.1 Factorial (!)

Definition 1.1.1 *Let n be a natural number. It is represented as $n!$ and it is read as n the multiplication factor.*

$$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3)..........3 \cdot 2 \cdot 1$$

Example 1.1.1

- $3! = 3 \cdot 2 \cdot 1 = 6$
- $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$
- $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

Property 1 *From the definition, it is concluded that:*

$$n! = n \cdot (n-1)! \tag{1.1}$$

Example 1.1.2

- $8! = 8 \cdot 7!$
- $5! = 5 \cdot 4!$
- $6! = 6 \cdot 5!$

The property is also used to simplify fractions.

Example 1.1.3

Simplify the following relationships:

- $$\frac{7!}{5!} = \frac{7 \cdot 6 \cdot 5!}{5!} = 7 \cdot 6 = 42$$

- $$\frac{n!}{(n-2)!} = \frac{n \cdot (n-1) \cdot (n-2)!}{(n-2)!} = n \cdot (n-1) = n^2 - n.$$

Property 2 *Let $a, b \in \mathbb{N}$, we have the following results:*

· $a! + b! \neq (a+b)!$

· $a! \times b! \neq (a \times b)!$

· $\frac{a!}{b!} \neq (\frac{a}{b})!$

· $(a!)^k \neq (a^k)!, \forall k \in \mathbb{N}$

Example 1.1.4 · $3! + 2! = 8 \neq (2+3)! = 120$

· $4! \times 5! = 2880 \neq (4 \times 5)! = 20!$

· $\frac{4!}{2!} = 12 \neq (\frac{4}{2})! = 2$

· $(2!)^3 = 8 \neq (2^3)! = 40320,$

1.2 Arrangements without repetition:

Definition 1.2.1 *Let E be a set with n element, and p is a natural number where :*

$$1 \leq p \leq n$$

We call A_n^p an arrangement of n elements taken without repetition and we define it as follows:

$$A_n^p = \frac{n!}{(n-p)!}$$

In addition, the arrangement is characterized by the importance of order and non-repetition.

Example 1.2.1

Let $E : \{a, b, c\}$

- How many subsets of E are there such that each set contains two consecutive elements?
- The arrangements of two elements from E are the sequenced subsets of two elements from E, and they are:

$$\{a, b\}, \{a, c\}, \{b, a\}, \{b, c\}, \{c, a\}, \{c, b\} = 6$$

- In another way,

$$A_3^2 = \frac{3!}{(3-2)!} = 6$$

1.3 Combinations without repetition

E is a set of n element and p is a natural number where : $p \leq n$. Every subset of E that includes p element is called a combination of p element from E.
The number of combinations is as follows:

$$C_n^p = \frac{n!}{p!(n-p)!}$$

Example 1.3.1

Let : $E = \{a, b, c\}$.

The Combinations of two elements from E are the subsets of two elements from E, and they are: {a, b}, {a, c}, {c, b} = 3 sets In another way :

$$C_2^3 = \frac{3!}{2!(3-2)!} = 3$$

Property 3 *Let p and n be two natural numbers, we have: For all $n \in \mathbb{N}$*

$$C_n^p = C_{n-1}^p + C_{n-1}^{p-1}, \qquad \forall 1 \leq k \leq n-1.$$

$$C_n^p = C_n^{n-p}, \quad \forall k \leq n.$$

$$C_n^n = C_n^0 = 1$$

1.4 Newton's Binomial Theorem

Definition 1.4.1 *For all $a, b \in \mathbb{R}$, For all $n \in \mathbb{N}^*$*

$$(a+b)^n = \sum_{p=0}^{n} C_n^p a^{n-p} b^p.$$

Example 1.4.1

Using Newton's law, develop the following formulas:$(a+b)^2, (x-2)^3$

- $$\begin{aligned}(a+b)^2 &= \sum_{p=0}^{2} C_2^p a^{2-p} b^p \\ &= C_2^0 a^{2-0} b^0 + C_2^1 a^{2-1} b^1 + C_2^2 a^{2-2} b^2 \\ &= C_2^0 a^2 + C_2^1 ab + C_2^2 b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

- $$\begin{aligned}(x-2)^3 &= \sum_{p=0}^{3} C_3^p x^{3-p} (-2)^p \\ &= C_3^0 x^{3-0}(-2)^0 + C_3^1 x^{3-1}(-2)^1 + C_3^2 x^{3-2}(-2)^2 + C_3^3 x^{3-3}(-2)^3 \\ &= C_3^0 x^3 + C_3^1 x^2(-2) + C_3^2 x(-2)^2 + C_3^3(-2)^3 \\ &= x^3 - 6x^2 + 12x - 8\end{aligned}$$