Solution of first series:

Exercise 01: simplify the relationships

$$\frac{13!}{11!} = \frac{13\times12\times11!}{11!} = 13\times12$$

$$\frac{3!15!}{12!} = \frac{3!\,15!}{12!} = \frac{3!\;15\times14\times13\times\cancel{12!}}{\cancel{12!}} = 3\times2\times1\times15\times14\times13$$

$$\frac{200!}{2!\,198!} = \frac{200\times199\times\cancel{198!}}{2\times\cancel{198!}} = \frac{200\times199}{2} =$$

$$\frac{5!+7!}{6!} = \frac{5!+7\times6\times5!}{6\times5!} = \frac{\cancel{5!}(1+7\times6)}{6\times\cancel{5!}} =$$

$$\frac{14!}{13!+12!} = \frac{14\times13\times12!}{13\times12!+12!} = \frac{14\times13\times\cancel{12!}}{\cancel{12!}(13+1)} = 13$$

$$\frac{(n+1)!}{n!} = \frac{(n+1)\,\cancel{n!}}{\cancel{n!}} = n+1$$

$$\frac{(n-1)!}{(n+1)!} = \frac{\cancel{(n-1)!}}{(n+1)n\,\cancel{(n-1)!}} = \frac{1}{n^2+n}$$

$$\frac{2n!}{(2n-3)!} = \frac{(2n)(2n-1)(2n-2)\cancel{(2n-3)!}}{\cancel{(2n-3)!}} = 2n(2n-1)(2n-2)$$

$$\frac{(n-1)!\;2!}{(n-2)!\;1!} = \frac{(n-1)\cancel{(n-2)!}\;2}{\cancel{(n-2)!}} = 2n-2$$

Exercise 02:

$$A_n^p = \frac{n!}{(n-p)!}$$

a-

$$A_4^3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 4\times3\times2\times1.$$

$$A_{10}^{10} = \frac{10!}{(10-10)!} = 10! = 10\times9\times\cdots\times1$$

$$C_3^2 = \frac{3!}{2!(3-2)!} = \frac{3!}{2!}$$

$$C_n^p = \frac{n!}{p!(n-p)!}$$

$$C_3^2 = 3$$

$$C_{10}^5 = \frac{10!}{5!(10-5)!} = \frac{10!}{5!\,5!} = \frac{10\times 9\times 8\times 7\times 6\times \cancel{5!}}{5!\,\cancel{5!}} =$$

(b) - prove That:

$$C_n^p = C_{n-1}^p + C_{n-1}^{p-1}$$

we have $C_{n-1}^p = \dfrac{(n-1)!}{p!(n-1-p)!}$, $C_{n-1}^{p-1} = \dfrac{(n-1)!}{(p-1)!(n-p)!}$

$$C_{n-1}^p + C_{n-1}^{p-1} = \frac{(n-1)!}{p!(n-1-p)!} + \frac{(n-1)!}{(p-1)!(n-p)!}$$

$$= \frac{(n-1)!\,(n-p)}{(n-p)\,p\,(p-1)!\,(n-1-p)!} + \frac{(n-1)!\,p}{p\,(p-1)!\,(n-p)\,(n-p-1)!}$$

$$= \frac{(n-1)!\,(n-p) + (n-1)!\,p}{p\,(p-1)!\,(n-p)\,(n-p-1)!}$$

$$= \frac{(n-1)!\,(n-p+p)}{p!\,(n-p)!}$$

$$= \frac{(n-1)!\,n}{p!\,(n-p)!} = \frac{n!}{p!\,(n-p)!} = C_n^p$$

Calculate C_5^2, C_6^1. from The above relation, we get

$$C_5^2 = C_{5-1}^2 + C_{5-1}^{2-1} = C_4^2 + C_4^1$$

$$C_6^1 = C_{6-1}^1 + C_{6-1}^{1-1} = C_5^1 + C_5^0$$

Exercise 03

- $A_n^2 - 20n = 0$, we have $A_n^2 = \frac{n!}{(n-2)!}$ so

$$\frac{n!}{(n-2)!} - 20n = 0$$

$$\Rightarrow \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} - 20n = 0$$

$$\Rightarrow n^2 - n - 20n = 0$$

$$\Rightarrow n^2 - 21n = 0$$

$$n(n-21) = 0$$

So, we have Two solutions:

$$\begin{cases} n = 0 \\ \text{or} \\ n - 21 = 0 \end{cases} \Rightarrow \begin{cases} n = 0 \\ \text{or} \\ n = 21 \end{cases}$$

$$S = \{0, 21\}$$

$\cdot\ C_n^1 + C_n^2 = 5n$

$$\frac{n!}{1!(n-1)!} + \frac{n!}{2!(n-2)!} = 5n$$

$$\frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} + \frac{n(n-1)\cancel{(n-2)!}}{2\cancel{(n-2)!}} = 5n$$

$$\frac{2n + n^2 - n}{2} = 5n$$

$$\Rightarrow n + n^2 = 10n$$

$$n^2 - 9n = 0 \Rightarrow n(n-9) = 0$$

we have Two, solutions

$$\begin{cases} n = 0 \\ \text{or} \\ n = 9 \end{cases}$$

$$S = \{0, 9\}$$

$\cdot\ 2A_n^2 + 50 = A_{2n}^2$

$$2\frac{n!}{(n-2)!} + 50 = \frac{(2n)!}{(2n-2)!}$$

$$2\left[\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}\right] + 50 = \frac{(2n)(2n-1)\cancel{(2n-2)!}}{\cancel{(2n-2)!}}$$

$$2n(n-1) + 50 = 2n(2n-1)$$

$$2n^2 - 2n + 50 = 4n^2 - 2n$$

$$2n^2 - \cancel{2n} + 50 - 4n^2 + \cancel{2n} = 0$$

$$-2n^2 + 50 = 0 \Rightarrow \begin{cases} n = 5 \checkmark \\ n = -5 \times \end{cases}$$

we have one solution $n = 5$ because $-5 \notin \mathbb{N}$

Exercise 04 :

Newton's Binomial Theorem:

$$(a+b)^n = \sum_{p=0}^{n} C_n^p \, a^{n-p} b^p \quad \forall a,b \in \mathbb{R}.$$

* $$(x-1)^3 = \sum_{p=0}^{3} C_3^p \, x^{3-p} (-1)^p$$

$$= C_3^0 x^{3-0}(-1)^0 + C_3^1 x^{3-1}(-1)^1 + C_3^2 x^{3-2}(-1)^2 + C_3^3 x^{3-3}(-1)^3$$

$$= C_3^0 x^3 - C_3^1 x^2 + C_3^2 x - C_3^3 \qquad \left\{ \begin{array}{l} C_3^0 = C_3^3 = 1 \\ C_3^1 = C_3^2 = 3 \end{array} \right.$$

$$= x^3 - 3x^2 + 3x - 1$$

* $$(x+y)^2 = \sum_{p=0}^{2} C_2^p \, x^{2-p} y^p$$

$$= C_2^0 x^{2-0} y^0 + C_2^1 x^{2-1} y^1 + C_2^2 x^{2-2} y^2$$

$$= C_2^0 x^2 + C_2^1 xy + C_2^2 y^2$$

$$= x^2 + 2xy + y^2$$

* $$(3x+2)^4 = \sum_{p=0}^{4} C_4^p \, (3x)^{4-p} (2)^p$$

$$= C_4^0 (3x)^{4-0}(2)^0 + C_4^1 (3x)^{4-1}(2)^1 + C_4^2 (3x)^{4-2}(2)^2 + C_4^3 (3x)^{4-3}(2)^3$$
$$+ C_4^4 (3x)^{4-4}(2)^4$$

$$= C_4^0 (3x)^4 + C_4^1 (3x)^3 (2) + C_4^2 (3x)^2 (4) + C_4^3 (3x)\, 8$$
$$+ C_4^4 \, 16$$

$$\left[\begin{array}{l} C_4^0 = 1 \\ C_4^1 = 4 \\ C_4^2 = 6 \\ C_4^3 = 4 \\ C_4^4 = 1 \end{array} \right.$$

$$= 81x^4 + 216x^3 + 216x^2 + 96x + 16$$

$$\left(x+\frac{1}{x}\right)^3 = \sum_{p=0}^{3} C_3^p\, x^{3-p}\left(\frac{1}{x}\right)^p$$

$$= C_3^0\, x^{3-0}\left(\frac{1}{x}\right)^0 + C_3^1\, x^{3-1}\left(\frac{1}{x}\right)^1 + C_3^2\, x^{3-2}\left(\frac{1}{x}\right)^2 + C_3^3\, x^{3-3}\left(\frac{1}{x}\right)^3$$

$$= C_3^0\, x^3 + C_3^1\, x^2\,\frac{1}{x} + C_3^2\, x\,\frac{1}{x^2} + C_3^3\,\frac{1}{x^3}$$

$$= x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$$

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