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Faculty of economic sciences, commerce and management sciences, First year, common trunk (2024-2025)
Second series (Numerical sequences).

Exercise 1 –

a- Let the sequence (u_n) defined by:

$$\begin{cases} u_0 = \frac{1}{2} \\ u_{n+1} = \frac{1}{2-u_n} \end{cases}$$

1. Calculate: u_1, u_2, u_3.
2. Prove by regression that: $\forall n \in \mathbb{N}, \quad u_n < 1$.
3. Prove that (u_n) is increasing.
4. Conclude with justification that (u_n) is convergent.

b- Let the sequence (v_n) defined by:

$$v_n = \frac{1}{1-u_n}$$

1. Calculate: v_0, v_1.
2. Prove that (v_n) is arithmetic sequence.
3. Express u_n and v_n in terms of n.
4. Calculate the sum $s_n = v_0 + v_1 + \cdots + v_n$

Exercise 2 –

Let the sequence (w_n) defined by the general term expression:

$$w_n = 2^n, \quad \forall n \in \mathbb{N}$$

1. Calculate : w_0, w_1, w_2.
2. Prove that (w_n) is a geometric sequence and determine its basis and first term.
3. Calculate the sum : $w_0 + w_1 + \cdots + w_n$.
4. Study the convergence of (w_n)

Exercise 3 –

On the first of January 2005, the population of a city is 100,000 people. We assume that the number of deaths is the number of births. Given its distinguished economic activity, 5,000 additional people settle in it annually. v_n represents its population on first of January, $2005 + n$.

1. What is the value of v_0, calculate v_1 and v_2.

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2. Express v_{n+1} in terms of v_n and then deduce the nature of the sequence (v_n).
3. What is the population of the city on January 1, 2023?
4. From which year did the city's population exceed 200,000 people.

Exercise 4 –

On January 1, 2001, Ahmed deposited 10,000 DZD in a bank that offered 5% interest annually. However, his transportation expenses to university required him to withdraw 1,500 DZD at the end of each year (after calculating interest). We denote by u_n Ahmed's balance on January 1, $2001 + n$.

a-
1. Express u_0 then u_1, how much balance did Ahmed have on January 1, 2003?
2. Prove thar for all $n \in \mathbb{N}$ we have, $U_{n+1} = 1.05u_n - 1500$

b- Let $v_n = u_n - 30000$

1. Prove that (v_n) is geometric sequence and write the express of v_n.
2. Calculate $\lim_{n\to+\infty} u_n$, what do you conclude.
3. Starting from which year does Ahmed's balance become a debt?