FACULTY OF ECONOMIC SCIENCES, COMMERCE AND MANAGEMENT SCIENCES, FIRST YEAR, COMMON TRUNK
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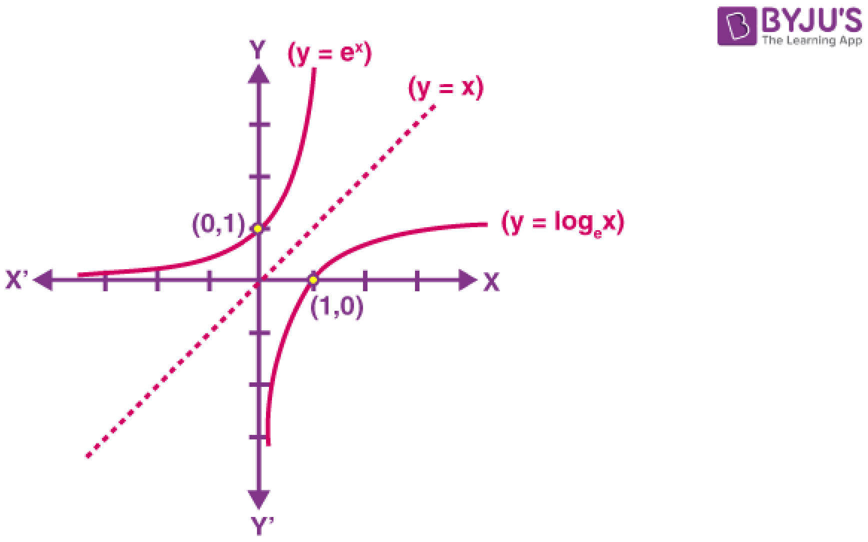
Chapter 03: Logarithmic and Exponential Functions



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Chapter 3

Logarithmic and Exponential Functions

3.1 Logarithmic Functions

Definition 3.1.1

- We call the logarithmic function the function that we denote by ln and which is associated with every real number x of the domain $]0, \infty[$ the number

$$\begin{aligned} \ln :\ ]0, \infty[&\to \mathbb{R} \\ x &\to \ln x \end{aligned}$$

- The logarithmic function $\ln x$ is defined if $x > 0$.
- The logarithmic function $\ln(f(x))$ is defined if $f(x) > 0$.

Example 3.1.1 *find the domain of definition of the following functions:*

- $f(x) = \ln(x^2 + x)$
- $f(x) = \ln(x + 2)$

The answer:

- *The function f is defined if $x + 2 > 0$, so $x > -2$, then $D_f =]-2, +\infty[$.*

- *The function f is defined if $x^2 + x > 0$, so we must study the sign of $x^2 + x$.*

$$x^2 + x = 0$$

$$\Delta = 1, x_1 = 0;\ x_2 = 1$$

		0		1	
$x^2 + x$	$+$	$\bullet$	$-$	$\bullet$	$+$

So $D_f =]-\infty, 0[\cup]1, +\infty[$.

3.2 Properties of logarithm functions

Let $a, b \in \mathbb{R}^+$, we have:

- $\ln(a \times b) = \ln a + \ln b \Longleftrightarrow \ln(4 \times 5) = \ln 4 + \ln 5$
- $\ln(\frac{a}{b}) = \ln a - \ln b \Longleftrightarrow \ln(\frac{8}{10}) = \ln 8 - \ln 10$
- $\ln(\frac{1}{a}) = -\ln a \Longleftrightarrow \ln(\frac{1}{10}) = -\ln 10$
- $\ln(a^n) = n \ln a \Longleftrightarrow \ln(4^5) = 5 \ln 4$
- $\ln 1 = 0$, $\ln e = 1$.

3.3 Derivation

Definition 3.3.1 *If $f(x)$ is differentiable on the domain $I \subset \mathbb{R}$, then for every number $x \in I$, we have*

$$[\ln(f(x))]' = \frac{(f(x))'}{f(x)}$$

3.4 Solving logarithmic equations and inequalities

3.4.1 Solving logarithmic equations

To solve an equation involving ln, we follow the following steps:

- Assign the domain of definition to the equation to be solved.
- We write the equation to be solved in one of the two forms:

$$\ln a = \ln b \Longleftrightarrow a = b$$
$$\ln a = k \Longleftrightarrow a = e^k$$

- We make sure that the solution belongs to the definition domain.

Example 3.4.1 *Solve the following equation:* $\ln(x+2) = 0$.

The answer: *The solution of* $\ln(x+2) = 0$, *as we saw in the previous example* $D_f =]-2, +\infty[$, *now we can write the equation as follows:*

$$\ln(x+2) = 0$$
$$x + 2 = e^0 = 1$$
$$x = -1$$

since $-1 \in]-2, +\infty[$*; so the solution is* δ*={-1}.*

3.4.2 Solving logarithmic inequalities

To solve inequalities involving ln, we follow the following steps:

- Assign the domain of definition to the inequality to be solved.
- We write the inequality to be solved in one of the following forms:

$$\ln a \leq \ln b \Longleftrightarrow a \leq b$$
$$\ln a \geq \ln b \Longleftrightarrow a \geq b$$
$$\ln a \leq k \Longleftrightarrow a \leq e^k$$
$$\ln a \geq k \Longleftrightarrow a \geq e^k$$

- Finally, we make an intersection between the definition domain and the second domain.

Example 3.4.2 *Solve the following inequalities:* $\ln(2x) < 1$ *and* $\ln(x) - 4 \geq 0$

The answer:

- *Solving* $\ln(2x) < 1$

  a- $\ln(2x)$ *is defined if* $2x > 0$*, so* $D_f =]0, +\infty[$

  b-

  $$\ln(2x) < 1 \Longrightarrow 2x < e^1 \Longrightarrow x < \frac{1}{2}e^1$$

  so we get $x \in]-\infty, \frac{1}{2}e^1[$,

  c- *The solution of the inequality is,* $\delta =]0, \frac{1}{2}e^1[$

- *Solving* $\ln(x) - 4 \geq 0$

  a- $\ln(x)$ *is defined if* $x > 0$*, so* $D_f =]0, +\infty[$

  b-

  $$\ln(x) - 4 \geq 0 \Longrightarrow \ln(x) \geq 4 \Longrightarrow x \geq ln4$$

  so we get $x \in [ln4, +\infty[$,

  c- *The solution of the inequality is,* $\delta =]0, ln4]$