# Solution of exercises serie n°: 1

**Exercise 01:**

* $3!\times(0!)^2 = (3\times2\times1)\times(1)^2 = 6\times1 = 6$

* $\frac{10!}{5!} = \frac{10\times9\times8\times7\times6\times\cancel{5!}}{\cancel{5!}} = 30240$ $\qquad / \; n! = n(n-1)!$

* $\frac{9!}{6!\times3!} = \frac{9\times8\times7\times\cancel{6!}}{\cancel{6!}\times3!} = \frac{9\times8\times7}{6} = \frac{504}{6} = 84$

* $\frac{7!\times\cancel{5!}}{\cancel{5!}} = 7! = 5040$

* $\frac{3!+4!}{2!} = \frac{3\times2!+4\times3\times2!}{2!} = \frac{\cancel{2!}(3+12)}{\cancel{2!}} = 15$

* $\frac{(n+1)!}{(n-1)!} = \frac{(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}} = n(n+1) = n^2+n$

* $\frac{(2n+1)!}{(2n-1)!} = \frac{(2n+1)(2n)\cancel{(2n-1)!}}{\cancel{(2n-1)!}} = (2n+1)(2n) = 4n^2+2n$

* $\frac{2n!-(2n-1)!}{2(n!)-(n-1)!} = \frac{(2n)(2n-1)!-(2n-1)!}{2\cdot(n-1)!-(n-1)!}$

  $= \frac{(2n-1)!\,\cancel{(2n-1)}}{(n-1)!\,\cancel{(2n-1)}} = \frac{(2n-1)!}{(n-1)!}$

* $\frac{3n!+(3n-2)!}{(3n-1)!} = \frac{3n(3n-1)\cancel{(3n-2)!}+\cancel{(3n-2)!}}{(3n-1)\cancel{(3n-2)!}}$

  $= \frac{3n(3n-1)+1}{3n-1}$

  $= \frac{9n^2-3n+1}{3n-1}$

**Exercise 02:** 1/ Reminder: $A_n^k = \frac{n!}{(n-k)!}$

$C_n^k = \frac{n!}{k!(n-k)!}$

* $A_5^2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5\times4\times\cancel{3!}}{\cancel{3!}} = 20$

* $A_3^3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3\times2\times\cancel{1}}{\cancel{1}} = 6$

* $C_6^2 = \frac{6!}{2!(6-2)!} = \frac{6\times5\times\cancel{4!}}{2\times\cancel{(4)!}} = \frac{30}{2} = 15$

* $C_4^3 = \frac{4!}{3!(4-3)!} = \frac{4\times\cancel{3!}}{\cancel{3!}\times1!} = 4$

2 / Proof =

$$* \; C^3_{n-1} + C^2_{n-1} = \frac{(n-1)!}{3!\,((n-1)-3)!} + \frac{(n-1)!}{2!\,((n-1)-2)!}$$

$$= \frac{(n-1)!}{3!\,(n-4)!} + \frac{(n-1)!}{2!\,(n-3)!}$$

$$= \frac{(n-1)(n-2)(n-3)\cancel{(n-4)!}}{3!\,\cancel{(n-4)!}} + \frac{(n-1)(n-2)\cancel{(n-3)!}}{2!\,\cancel{(n-3)!}}$$

$$= \frac{(n-1)(n-2)(n-3)}{3 \times 2!} + \frac{(n-1)(n-2)}{2!}$$

$$= \frac{(n-1)(n-2)}{2!}\left(\frac{n-3}{3} + 1\right)$$

$$= \frac{(n-1)(n-2)}{2!}\left(\frac{n \cancel{-3} \cancel{+3}}{3}\right)$$

$$= \frac{n(n-1)(n-2)}{3 \times 2!} = \frac{n(n-1)(n-2)}{3!}$$

$$= \frac{n(n-1)(n-2)(n-3)!}{3!\,(n-3)!} = \frac{n!}{3!\,(n-3)!} = \boxed{C^3_n} \; \#$$

$$* \; C^{n-2}_n = \frac{n!}{(n-2)!\,(n-(n-2))!}$$

$$= \frac{n!}{(n-2)!\,(\cancel{n} - \cancel{n} + 2)!} = \frac{n!}{2!\,(n-2)!} = C^2_n \; \#$$

$$* \; \frac{n!}{2!\,(n-2)!} = \frac{1}{2!}\left(\frac{n!}{(n-2)!}\right)$$ / We have $A^k_n = \frac{n!}{(n-k)!}$

$$= \frac{1}{2!}\left(A^2_n\right) = \frac{A^2_n}{2!} \; \#$$ so $A^2_n = \frac{n!}{(n-2)!}$

$$* \; p\, C^p_{n+1} = p\left(\frac{(n+1)!}{p!\,(n+1-p)!}\right)$$

$$= \frac{\cancel{p}(n+1)(n)!}{\cancel{p}(p-1)!\,(n-(p-1))!} = (n+1)\left(\frac{n!}{(p-1)!\,(n-(p-1))!}\right)$$

$$= (n+1)\, C^{p-1}_n \; \#$$

$n^4$

Exercise 03 = Determination of the natural number "n"

$$* \quad C_n^1 + C_n^2 = \frac{n!}{1!(n-1)!} + \frac{n!}{2!(n-2)!}$$

$$= \frac{n(\cancel{n-1})!}{(\cancel{n-1})!} + \frac{n(n-1)(\cancel{n-2})!}{2(\cancel{n-2})!}$$

$$= n + \frac{n(n-1)}{2}$$

$$= \frac{2n+n^2-n}{2} = \frac{n^2+n}{2} = 10$$

$$n^2+n=20 \Rightarrow n^2+n-20=0$$

$$\Delta = b^2-4ac$$

$$= 1-4(1)(-20) = 81$$

$$n_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-1-\sqrt{81}}{2(1)} = \frac{-1-9}{2} = \frac{-10}{2} = -5 \text{ , (Unaccepted)}$$

$$n_2 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-1+\sqrt{81}}{2(1)} = \frac{-1+9}{2}$$

$$= \frac{8}{2} = 4$$

$$\boxed{n=4}$$

$$* \quad 2A_n^2 + 50 = A_{2n}^2$$

$$2\left(\frac{n!}{(n-2)!}\right) + 50 = \frac{(2n)!}{(2n-2)!}$$

$$\Rightarrow 2\left(\frac{n(n-1)(\cancel{n-2})!}{(\cancel{n-2})!}\right) + 50 = \frac{(2n)(2n-1)(\cancel{2n-2})!}{(\cancel{2n-2})!}$$

$$\Rightarrow 2n(n-1)+50 = 2n(2n-1)$$

$$2n^2-2n+50 = 4n^2-2n$$

$$4n^2-2n^2-\cancel{2n}+\cancel{2n}-50=0$$

$$2n^2-50=0$$

$$2n^2=50$$

$$n^2 = \frac{50}{2} = 25$$

$$n = \begin{cases} -5 & \text{Unaccepted} \\ \boxed{5} & \text{Accepted} \end{cases}$$

$$\boxed{n=5}$$

* $\frac{2n!}{(2n-2)!} + 2n = 4$

$$\frac{2n(2n-1)\cancel{(2n-2)!}}{\cancel{(2n-2)!}} + 2n = 4$$

$$2n(2n-1) + 2n = 4$$

$$4n^2 - \cancel{2n} + \cancel{2n} = 4$$

$$n^2 = \frac{4}{4} = 1$$

$$n = \begin{cases} -1 & \text{Unaccepted} \\ 1 & \text{accepted.} \end{cases}$$

$$\boxed{n = 1}$$

Exercise 04 * 1/ If we select 4 maths from 6 different maths books

- We note that we are faced with the following conditions.
  * The part of the whole (4 from 6)
  * Order is not important.
  * Repetition is not allowed.

So we use (r-combination)

- The same for the the english books.

We notice that the seven books are arranged so we multiply the result by 7!.

$$C_6^4 \times C_5^3 \times 7! = \frac{6!}{4!(6-4)!} \times \frac{5!}{3!(5-3)!} \times 7!$$

$$= \frac{6 \times 5 \times \cancel{4!}}{\cancel{4!}\ 2!} \times \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}\ 2!} \times 7!$$

$$= \frac{30}{2} \times \frac{20}{2} \times 7!$$

$$= 15 \times 10 \times 5040 = \boxed{756000}$$

* If the 4 maths books remain together, we note that the order is important (it is arranged in order), we multiply $A_6^4$ by 4!

So we use (r-permutations) for the maths books and we use (r-combinations) for the english books.

$$4! \times P_6^4 \times C_5^3 = 4! \times \frac{6!}{(6-4)!} \times \frac{5!}{3!(5-3)!}$$

$$= 4! \times \frac{6!}{2!} \times \frac{5!}{3!(2)!}$$

$$= 4! \times \frac{6 \times 5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}} \times \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!} \times 2!}$$

$$= 24 \times 360 \times 10 = \boxed{86400}$$

Excercise 05.

1 - The $1^{st}$ situation: We are faced with the following conditions:

- The part of the whole (4 of 8 books)
- The order is not important.
- Repetition is not allowed

So we use (K- combinations)

$$C_8^4 = \frac{8!}{4!.(8-4)!} = \frac{8!}{4!.4!} = \frac{8 \times 7 \times 6 \times 5 \times \cancel{4!}}{4! \times \cancel{4!}}$$

$$= \frac{1680}{24} = \boxed{70}$$

2 / The $2^{nd}$ situation. We are faced with the following conditions

* The part of the whole (3 of 10 competitors)
* The order is important.
* Repetition is not allowed. So we use (K - [illegible]ations)

$$P_{10}^3 = \frac{10!}{(10-3)!} = \frac{10 \times 9 \times 8 \times \cancel{7!}}{\cancel{7!}} = 720 \quad \boxed{720}$$

3 / The $3^{rd}$ situation: to choose a committee (men / women) We are faced with the following conditions

* The part of the whole.
* The order is not important.
* Repetition is not allowed; So we use (K-Combinations)

$$C_{10}^5 \times C_{12}^6 = \frac{10!}{5!(10-5)!} \times \frac{12!}{6!(12-6)!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times \cancel{5!}}{5!. \times \cancel{5!}} \times \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times \cancel{6!}}{6! \times \cancel{6!}}$$

$= 252 \times 924$

$= \boxed{232848}$

* The 4th situation, We are faced the conditions:
  * The whole elements (permutations where $k = n$)
  * The order is important.
  * and there are a repeated elements.

So we use the following rule. "s" is repeated 3 times: $n_1 = 3$; "e" is repeated 2 times: $n_2 = 2$

$$P_n^{n_1, n_2} = \frac{n!}{n_1!\, n_2!} \Rightarrow P_9^{3,2} = \frac{9!}{3!\,2!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!} \times 2!}$$

$$= 30240$$

Exercise 06 = binomial theorem

$$(a+b)^n = \sum_{k=0}^{n} C_n^k a^k b^{n-k}$$

* $(1+x)^4 = \sum_{k=0}^{4} C_4^k a^k b^{4-k}$

$= C_4^0 1^0 x^4 + C_4^1 1^1 x^3 + C_4^2 1^2 x^2 + C_4^3 1^3 x^1 + C_4^4 1^4 x^0$

$= x^4 + 4x^3 + 6x^2 + 4x + 1$

* $(2+x)^3 = \sum_{k=0}^{3} C_3^k 2^k x^{3-k}$

$= C_3^0 2^0 x^3 + C_3^1 2^1 x^2 + C_3^2 2^2 x + C_3^3 2^3 x^0$

$= x^3 + 6x^2 + 12x + 8$

* $(1-3x)^2 = \sum_{k=0}^{2} C_2^k 1^k (-3x)^{2-k}$

$= C_2^0 1^0 (-3x)^2 + C_2^1 1^1 (-3x)^1 + C_2^2 1^2 (-3x)^0$

$= 9x^2 - 6x + 1$

* $(a-b)^7 = \sum_{k=0}^{7} C_7^k a^k (-b)^{7-k} = C_7^0 a^0 (-b)^7 + C_7^1 a(-b)^6 + C_7^2 a^2(-b)^5 + C_7^3 a^3(-b)^4$
$+ C_7^4 a^4(-b)^3 + C_7^5 a^5(-b)^2 + C_7^6 a^6(-b)^1 + C_7^7 a^7(-b)^0$

$= -b^7 + 7ab^6 - 21a^2b^5 + 35a^3b^4 - 35a^4b^3 + 21a^5b^2 - 7a^6b + a^7$

2/ The coefficient of $x^5$

$(1+4x)^9 = \sum_{k=0}^{9} C_9^k 1^k (4x)^{9-k} = C_9^0 1^0 (4x)^9 + C_9^1 1^1 (4x)^8 + C_9^2 1^2 (4x)^7 + C_9^3 1^3 (4x)^6$
$+ C_9^4 1^4 (4x)^5 + C_9^5 1^5 (4x)^4 + C_9^6 1^6 (4x)^3 + C_9^7 1^7 (4x)^2 + C_9^8 1^8 (4x) + C_9^9 1^9 (4x)^0$

$= 262144 x^9 + 589824 x^8 + 589824 x^7 + 344064 x^6 + 129024 x^5 + 32256 x^4$
$+ 5376 x^3 + 576 x^2 + 36x + 1$

$\boxed{\text{coeff } x^5 = 129024}$