Solution of exercises serie n°=2

Exercise 1-

1/ Determination of the first - 4 terms

$U_n = 2n^2 - n + 3$

$U_0 = 2(0)^2 - 0 + 3 = 0 - 0 + 3 = 3$

$U_1 = 2(1)^2 - 1 + 3 = 2 - 1 + 3 = 4$

$U_2 = 2(2)^2 - 2 + 3 = 8 - 2 + 3 = 9$

$U_3 = 2(3)^2 - 3 + 3 = 18$

$V_n = \dfrac{3n+5}{2-3n}$

$V_0 = \dfrac{3(0)+5}{2-3(0)} = \dfrac{5}{2}$

$V_1 = \dfrac{3(1)+5}{2-3(1)} = \dfrac{8}{-1} = -8$

$V_2 = \dfrac{3(2)+5}{2-3(2)} = \dfrac{11}{-4} = -\dfrac{11}{4}$

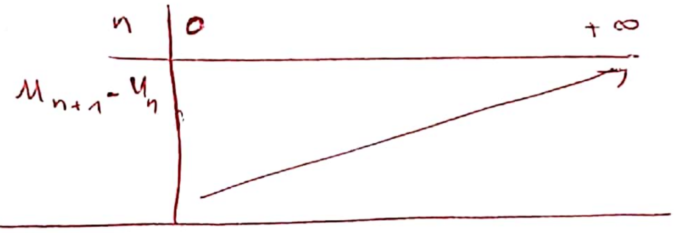
$V_3 = \dfrac{3(3)+5}{2-3(3)} = \dfrac{14}{-7} = \boxed{-2}$

1-1/ $U_{n+1} - U_n$:

$U_{n+1} = 2(n+1)^2 - (n+1) + 3 = 2(n^2 + 2n + 1) - n - 1 + 3$
$= 2n^2 + 4n + 2 - n + 2 = 2n^2 + 3n + 4$

$U_{n+1} - U_n = 2n^2 + 3n + 4 - (2n^2 - n + 3) = \cancel{2n^2} + 3n + 4 - \cancel{2n^2} + n - 3 = 4n + 1$

$4n + 1 = 0 \Rightarrow n = -\dfrac{1}{4}$ refused then $U_{n+1} - U_n > 0$



We conclude that U_n is increasing

2 / $\begin{cases} U_0 = 1 \\ U_{n+1} = U_n - 7 \end{cases}$ $\forall n \in \mathbb{N}$

$U_1 = U_0 - 7 = 1 - 7 = -6$

$U_2 = U_1 - 7 = -6 - 7 = -13$

$U_3 = U_2 - 7 = -13 - 7 = -20$

2.2/ $U_{n+1} - U_n = U_n - 7 - U_n = -7 < 0$ We conllude that U_n is decreasing.

Exercice n°= 2 Study of the direction:

1/ $U_n = 2n+1$

$U_{n+1} - U_n = 2(n+1)+1-(2n+1) = \cancel{2n}+2+\cancel{1}-\cancel{2n}-\cancel{1} = 2 > 0$

U_n is increasing for all $n \geq 0$

2/ $V_n = 5 \times 3^n$.

$V_{n+1} - V_n = 5 \times 3^{n+1} - 5 \times 3^n = 5 \times 3^n \times 3 - 5 \times 3^n = 5 \times 3^n \times (3-1)$
$= 2 \times 5 \times 3^n = 10 \times 3^n > 0$

So V_n is increasing.

3/ $W_n = 4 \times 3^n + 9$

$W_{n+1} - W_n = 4 \times 3^{n+1} + \cancel{9} - 4 \times 3^n - \cancel{9} = 4 \times 3^n \times 3 - 4 \times 3^n$
$= 4 \times 3^n (3-1)$
$= 8 \times 3^n > 0$

So W_n is increasing.

Exercice 03 = $(U_n)_{n \in \mathbb{N}}$ arithmetic sequence, $r = -2$

$U_0 = 15$.

1/ $U_{n+1} = U_n - 2$

2/ $U_n = U_0 + nr = 15 - 2n$

3/ $U_1 = 15 - 2(1) = 15 - 2 = 13$

$U_{10} = 15 - 2(10) = 15 - 20 = -5$

4/ Calculation of the sum $S_{10} = U_0 + U_1 + \dots + U_{10}$

$S = \frac{\text{element number}}{2}(\text{first term} + \text{last term})$

$= \frac{10-0+1}{2}(U_0 + U_{10})$

$= \frac{11}{2}(15+(-5)) = \frac{11}{2}(10) = 55$

Exercise 04 :

$(U_n)_{n\in\mathbb{N}}$ a geometric sequence. $U_1 = \frac{3}{2}$ / $U_4 = \frac{3}{16}$

1/ Determination of the reason "q" :

we have : $U_l = U_p \times q^{l-p}$

$$U_4 = U_1 \times q^{4-1}$$

$$\frac{3}{16} = \frac{3}{2} \times q^3$$

$$q^3 = \frac{3}{16} \div \frac{3}{2} = \frac{\cancel{3}}{16} \times \frac{2}{\cancel{3}} = \frac{2}{16} = \frac{1}{8}$$

$$q = \left(\frac{1}{8}\right)^{\frac{1}{3}} = \boxed{\frac{1}{2}}$$

2/ $U_n = U_0 \times q^n = U_1 \times q^{n-1}$

$$= \frac{3}{2} \times \left(\frac{1}{2}\right)^{n-1} = \frac{3}{2} \times \left(\frac{1}{2}\right)^n \times \left(\frac{1}{2}\right)^{-1}$$

$$= \frac{3}{2} \times \frac{\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$$

$$= \frac{3}{\cancel{2}} \times \left(\frac{1}{2}\right)^n \times \frac{\cancel{2}}{1}$$

$$= \boxed{3 \times \left(\frac{1}{2}\right)^n} \Rightarrow U_0 = 3\left(\frac{1}{2}\right)^0 = 3$$

3/ Calculation of the sum $S_{15} = U_0 + U_1 + \dots + U_{15}$

$$S = U_0\left(\frac{1 - q^{(15-0+1)}}{1-q}\right)$$

$$= 3\left(\frac{1 - \left(\frac{1}{2}\right)^{16}}{1 - \frac{1}{2}}\right)$$

$$= 3\left(\frac{1 - \left(\frac{1}{2}\right)^{16}}{\frac{1}{2}}\right) = 3 \times \frac{2}{1}\left(1 - \left(\frac{1}{2}\right)^{16}\right)$$

$$= 6 - 6\left(\frac{1}{2}\right)^{16} \simeq 5{,}999 \simeq 6$$

Exercice 05

$$U_n = \frac{3\times 2^n - 4n - 3}{2} \quad / \quad V_n = \frac{3\times 2^n + 4n + 3}{2}$$

1/ $W_n = U_n + V_n$.

$$= \frac{3\times 2^n - 4n - 3}{2} + \frac{3\times 2^n + 4n + 3}{2}$$

$$= 3\times 2^n \cancel{-4n} \cancel{-3} + 3\times 2^n + \cancel{4n} \cancel{+3}$$

$$= \frac{6\times 2^n - 0}{2} = 3\times 2^n \cancel{+3}$$

$$W_{n+1} = 3\times 2^{n+1} \cancel{+3} = \cancel{3\times 2^n \times 2 - 3}$$

$$= \cancel{3(2^{n+1} - 1)}$$

$$= 2(3\times 2^n) = 2W_n$$

Then W_n is a geometric sequence with $q = 2$

2/ $T_n = U_n - V_n$.

$$= \frac{3\times 2^n - 4n - 3}{2} - \frac{3\times 2^n + 4n + 3}{2}$$

$$= \frac{\cancel{3\times 2^n} - 4n - 3 - \cancel{3\times 2^n} - 4n - 3}{2}$$

$$= \frac{-8n - 6}{2} = -4n - 3$$

$$T_{n+1} = -4(n+1) - 3 = -4n - 4 - 3 = (-4n - 3) + (-4)$$

$$= T_n + r$$

So T_n is an arithmetic sequence with $r = -4$

3/ Proof that $U_n = \frac{W_n + T_n}{2}$

$$\frac{W_n + T_n}{2} = \frac{3\times 2^n - 4n - 3}{2} = U_n.$$

Exercise 06 $(U_n)_{n\in\mathbb{N}}$ an arithmetic sequence. $U_1 = 3$ / $U_3 = 9$

$$U_q = U_p + (q - p)r.$$

$$U_3 = U_1 + (3-1)r \Rightarrow 9 = 3 + 2r$$

$$2r = 9 - 3 = 6 \Rightarrow r = 3$$

$$U_n = U_1 + (n-1)r = 3 + (n-1)3 = \cancel{3} + 3n \cancel{-3} = 3n$$

$$U_5 = 3(5) = 15 \quad / \quad U_{20} = 3(20) = 60$$

$$S_{20} = U_5 + U_6 + \dots + U_{20}$$

$$= \frac{(20 - 5 + 1)}{2}(U_5 + U_{20}) = \frac{16}{2}(15 + 60) = 8\times 75$$

$$= \boxed{600}$$