NONLINEAR ELLIPTIC SYSTEMS INVOLVING $(p(x), q(x))$ −LAPLACIAN

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Abstract: In this talk, by using the mountain pass theorem, we obtain the existence of non trivial weak solutions of the following nonlocal elliptic system

$$\begin{cases} -M_1\Big(\displaystyle\int_\Omega \frac{1}{p(x)}|\Delta u|^{p(x)}dx\Big)\Delta(|\Delta u|^{p(x)-2}\Delta u) = F_u(x,u,v) & \text{in } \mathbb{R}^N, \\ -M_2\Big(\displaystyle\int_\Omega \frac{1}{q(x)}|\Delta v|^{q(x)}dx\Big)\Delta(|\Delta v|^{q(x)-2}\Delta v) = F_v(x,u,v) & \text{in } \mathbb{R}^N, \end{cases} \tag{1}$$

p and q are real valued functions satisfying $1 < p(x), q(x) < N \quad (N \geq 2)$ for every $x \in \mathbb{R}^N$, and M_1 and M_2 are continuous and bounded functions. The real valued function $F \in C^1(\mathbb{R}^N \times \mathbb{R}^2)$ satisfies some assumptions. The unknown real valued functions u and v stay in appropriate spaces. The operator $\Delta_{p(x)}u = div\left(|\nabla u|^{p(x)-2}\nabla u\right)$ designates the $p(x)$-Laplacian.

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