New Approach for the Computation of the Water Surface Angle in Partially Filled Pipes: Pipes Arranged in Parallel

Lotfi Zeghadnia[1] and Jean Loup Robert[2]

Abstract: This article presents a new approach for the computation of the water surface angle in pipes arranged in parallel. Estimating this parameter in partially filled pipes is an important task in the solution of many practical problems in the different branches of the engineering profession such as flow measurement and design of drainage networks, where the flow is mostly of the free surface type. This characteristic is needed in the trial and error classical solution and huge number of times. The new method is elaborated to overcome the laborious trial and error method. Here, the computation of the water surface angle becomes easy, simple, and direct. **DOI: 10.1061/(ASCE)PS.1949-1204.0000272.** © *2017 American Society of Civil Engineers.*

Author keywords: Water surface angle; Free surface flow; Uniform and steady flow; Circular pipe; Manning equation.

Introduction

Most free surface flow calculation assumes the flow to be uniform and steady to simplify the computation of its parameters. Among these parameters is the water surface angle. These characteristics are important in engineering practice. Sewer and drainage systems are frequently assumed to flow with free surfaces. The Manning equation for free surface flow is considered the best model to describe this type of flow (Saâtçi 1990; Giroud et al. 2000; Akgiray 2004, 2005; Zeghadnia et al. 2014a, b). Several researchers have discussed this model such as Chow (1959), Henderson (1966), Metcalf and Eddy (1981), Carlier (1980), and Hager (2010). Circular shape is the preferred cross section form for sewer system design.

Sewersheds can be arranged in series or in parallel; similarly, pipes can be arranged in series or in parallel. Using the Manning equation, the computation of the water surface angle is not direct and has to go through a trial and error method with heavy computation. Based on the Manning model, a number of researchers have tried to propose an explicit solution for free surface flow computation. Among these are Saâtçi (1990), Giroud et al. (2000), and (Akgiray 2004, 2005). They tried to eliminate the need for trial and error methods for water surface angle ranging between 0° and 302.41°. Other authors have used the Colebrook-White model, including Prabhata (1994), Prabhata and Pushpa (2004), and Achour and Bedjaoui (2006).

In this study, using the Manning model, the authors propose a new approach, which is much simpler and more accurate than existing methods for the computation of the water surface angle for partially filled pipes arranged in parallel, for all the range of surface water angles from 0° to 360°.

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Note. This manuscript was submitted on May 7, 2015; approved on February 15, 2017; published online on May 11, 2017. Discussion period open until October 11, 2017; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Pipeline Systems Engineering and Practice***, © ASCE, ISSN 1949-1190.

Manning Equation

This paper discusses sewerage systems involving circular sections that flow partially full. The Manning equation (Manning 1891) has been widely used to compute free surface uniform flow, which indicates the flow must be steady and uniform, where the slope, cross-sectional flow area, and velocity are time independant, and are constant along the pipe length (Carlier 1980). Graphs and tables (Camp 1946; Swarna and Modak 1990) are established to facilitate the application of the Manning equation for the estimation of flow characteristics (Terence 1991). The Manning equation can be written as follows:

$$Q = \frac{1}{n} R_h^{2/3} A S^{1/2} \tag{1}$$

$$V = \frac{1}{n} R_h^{2/3} S^{1/2} \tag{2}$$

where Q = flow rate (m^3/s); R_h = hydraulic radius (m), (which is defined as the ratio of the channel's cross-sectional area of the flow to its wetted perimeter); n = pipe roughness coefficient (Manning n); A = cross sectional flow area (m^2); S = pipe bottom slope, dimensionless; and V = flow velocity (m/s).

Eqs. (1) and (2) can be rewritten in terms of the water surface angle of the pipe as shown in Fig. 1 as follows:

$$Q = \frac{1}{n}\left(\frac{D^8}{2^{13}}\right)^{1/3}\left[\frac{(\theta - \sin\theta)^5}{\theta^2}\right]^{1/3} s^{1/2} \tag{3}$$

$$V = \frac{1}{n}\left(\frac{D}{4}\right)^{2/3}\left[\frac{(\theta - \sin\theta)}{\theta}\right]^{2/3} s^{1/2} \tag{4}$$

$$A = \frac{D^2}{8}[\theta - \mathrm{Sin}(\theta)] \tag{5}$$

$$P = \theta\frac{D}{2} \tag{6}$$

$$R_h = \frac{A}{P} = \frac{D}{4}\left[1 - \frac{\sin(\theta)}{\theta}\right] \tag{7}$$

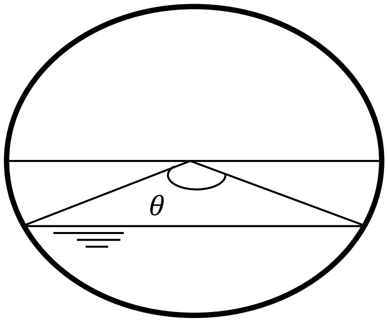

Fig. 1. Water surface angle

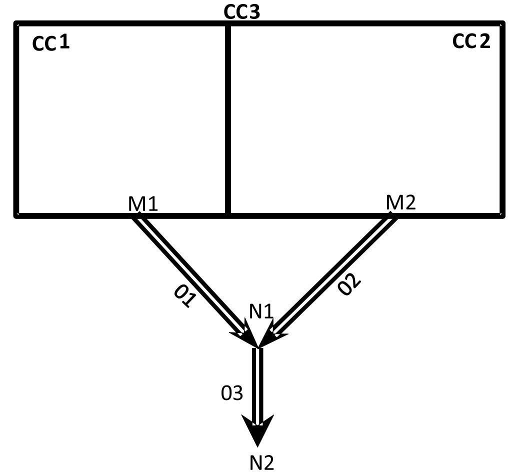

Fig. 2. Subwatersheds and pipes arranged in parallel

where D = pipe diameter (m); P = wetted perimeter (m); and θ = water surface angle (radian).

In the equations mentioned above, the computation of the water surface angle is not direct and requires an iterative procedure. A number of authors tried to propose an explicit formula for the computation of the water surface angle. Among these, the authors particularly note the work of Saatçi (1990) and Akgiray (2005). Saatçi (1990) has proposed an approximate equation to determine θ to avoid the need for the trial and error methods using Eq. (8):

$$\theta_{\text{Saatçi}} = \frac{3\pi}{2}\sqrt{1-\sqrt{1-\sqrt{\frac{\pi Q n}{D^{8/3}S^{0.5}}}}} \tag{8}$$

The Saatçi (1990) equations can only be used for water surface angles that are smaller than 265° (Saatçi 1990).

Akgiray (2005) tried to improve the approaches proposed earlier and proposed an explicit approximate solution for two cases: when the Manning coefficient n is constant (which is a common and conventional practice), and when n varies with the flow depth as documented by Camp (1946). In both cases, n is assumed to be known. In this research, the authors are interested in the first problem. For this particular case, Akgiray (2005) has proposed the following equation to estimate the water surface angle:

$$\theta = 2\times 6^{5/13}K^{3/13}\{1+[\sin^{-1}(2.98K)]^{0.8}-2K^{0.946}\} \tag{9}$$

where

$$K = \frac{Qn}{D^{8/3}S^{0.5}} \tag{10}$$

Eq. (9) is valid for θ between 0° and 301.41° where the maximum error equals 0.72%. The flow velocity can be estimated using Eqs. (4) and (9) with maximum deviation of 0.29%.

Analytical Formulation

Sewersheds may be arranged in series (Zeghadnia et al. 2014b) or in parallel. In this study, the authors focus on the second case, where the pipes are arranged in parallel as shown in Fig. 2:

- Pipe M1-N1 collects water from sewershed CC1, which takes on number 1;
- Pipe M2-N1 collects water from the equivalent sewershed CC2, which takes on number 2; and
- Pipe N1-N2 collects water from the equivalent sewershed CC3, which takes on number 3. Therefore

$$CC_{3=}CC_2 + CC_1 \tag{11}$$

The flow Q can be estimated using traditional methods such as the rational or other methods (Viessman and Lewis 2003). Four scenarios can be found for the computation of the water surface angle using the concept of a reference. A reference pipe is a pipe with known flow characteristics. The scenarios are as follow:

1. The computation of θ_3 as a function of the pipe 01 characteristics (pipe 01 is the reference pipe).
2. The computation of θ_3 as a function of the pipe 02 characteristics (pipe 02 is the reference pipe).
3. The computation of θ_2 as a function of the pipe 01 characteristics (pipe 1 is the reference pipe).
4. The computation of θ_1 as a function of the pipe 02 characteristics (pipe 2 is the reference pipe).

Cases One and Two

Let us consider that the pipe M2-N1, pipe 02, is the reference pipe with known parameters. Therefore, diameter D2, hydraulic radius Rh2, surface water angle θ2, water cross section A2, and slope S2 are known data. The slope S3 and roughness n3 are considered to be known parameters for pipe N1-N2, or pipe 03.

Q_1 is produced in sewershed CC1 and transported in pipe M1-N1, Q_2 is produced in sewershed CC2 and transported in pipe M2-N1, and Q_3 is produced in sewershed CC3 and transported in pipe N1-N2.

Eq. (4) can be written as follows (Zeghadnia et al. 2009):

$$V = \left[\left(\frac{S^{1/2}}{n}\right)^3\left(\frac{2Q}{D}\right)^2\right]^{1/5}\theta^{-2/5} \tag{12}$$

In the case of a partially filled pipe and according to Eq. (12), the previous formula can be rewritten as follows, where pipe 02 is the reference pipe (Zeghadnia et al. 2014a):

$$V_3 = \left(\frac{Q_3}{Q_2}\right)^{1/4}\left(\frac{S_3^{0.5}}{n_3}\right)^{3/4}\left(\frac{n_2}{S_2^{0.5}}\right)^{3/20}\left(\frac{2Q_2}{D_2\theta_2}\right)^{2/5} \tag{13}$$

Similarly, in the case where pipe M1-N1 is the reference pipe, the flow velocity can be calculated as follows (Zeghadnia et al. 2014a):

$$V_3 = \left(\frac{Q_3}{Q_1}\right)^{1/4}\left(\frac{S_3^{0.5}}{n_3}\right)^{3/4}\left(\frac{n_1}{S_1^{0.5}}\right)^{3/20}\left(\frac{2Q_1}{D_1\theta_1}\right)^{2/5} \tag{14}$$

Eq. (15) can be obtained using Eq. (12) as follows:

$$\theta = \left[\left(\frac{S^{0.5}}{n}\right)^3 \left(\frac{2Q}{D}\right)^2\right]^{1/2} \frac{1}{V^{5/2}} \tag{15}$$

Based on Eqs. (15) and (13), one can deduce the water surface angle of pipe 03 as a function of the known characteristics of the reference pipe 02 as follows:

$$\theta_3 = \left(\frac{n_3}{S_3^{0.5}}\right)^{3/8} \left(\frac{Q_2}{Q_3}\right)^{5/8} \left(\frac{2Q_3}{D_3}\right) \left(\frac{D_2\theta_2}{2Q_2}\right) \left(\frac{S_2^{0.5}}{n_2}\right)^{3/8} \tag{16}$$

From Eq. (16), the computation of the water surface angle becomes direct, easy, and simpler and eliminates the need for calculating of θ_3 iteratively. It produces the exact values of the water surface angle in pipe 03 as a function of the reference pipe 02 characteristics. Similarly, for the case when pipe 01 is the reference pipe, the expression of the water surface angle θ_3 can be written as follows:

$$\theta_3 = \left(\frac{n_3}{S_3^{0.5}}\right)^{3/8} \left(\frac{Q_1}{Q_3}\right)^{5/8} \left(\frac{2Q_3}{D_3}\right) \left(\frac{D_1\theta_1}{2Q_1}\right) \left(\frac{S_1^{0.5}}{n_1}\right)^{3/8} \tag{17}$$

Cases Three and Four

Here, the characteristics of the first pipe using the second pipe characteristics (reference pipe) will be shown in the following sections. For the case of a partially full pipe, and according to Eq. (12), the flow velocity can be calculated as follows (Zeghadnia et al. 2014a):

$$V_1 = \left(\frac{Q_1}{Q_2}\right)^{1/4} \left(\frac{S_1^{0.5}}{n_1}\right)^{3/4} \left(\frac{n_2}{S_2^{0.5}}\right)^{3/20} \left(\frac{2Q_2}{D_2\theta_2}\right)^{2/5} \tag{18}$$

For the case when pipe 01 is the reference pipe, the same results can be obtained as follows:

$$V_2 = \left(\frac{Q_2}{Q_1}\right)^{1/4} \left(\frac{S_2^{0.5}}{n_2}\right)^{3/4} \left(\frac{n_1}{S_1^{0.5}}\right)^{3/20} \left(\frac{2Q_1}{D_1\theta_1}\right)^{2/5} \tag{19}$$

Using Eqs. (15) and (18), one can deduce the water surface angle of pipe 01 as a function of the known characteristics of the reference pipe 02 as follows:

$$\theta_1 = \left(\frac{n_1}{S_1^{0.5}}\right)^{3/8} \left(\frac{Q_2}{Q_1}\right)^{5/8} \left(\frac{2Q_1}{D_1}\right) \left(\frac{D_2\theta_2}{2Q_2}\right) \left(\frac{S_2^{0.5}}{n_2}\right)^{3/8} \tag{20}$$

Similarly, for the case when pipe 01 is the reference pipe, the expression of the water surface angle θ_2 in pipe 02 can be written as follows:

$$\theta_2 = \left(\frac{n_2}{S_2^{0.5}}\right)^{3/8} \left(\frac{Q_1}{Q_2}\right)^{5/8} \left(\frac{2Q_2}{D_2}\right) \left(\frac{D_1\theta_1}{2Q_1}\right) \left(\frac{S_1^{0.5}}{n_1}\right)^{3/8} \tag{21}$$

Eqs. (20) and (21) produce the exact values of the water surface angle in pipe 01 as a function of the reference pipe 02 characteristics, or in pipe 02 as a function of the reference pipe 01 characteristics, where the computation become easy and faster.

Accuracy Test Method

To calculate the values of each column in table 01, one should apply following steps:

Step 1: Take in consideration the entire range of theta θ between 0° and 360°.

Step 2: Using Eq. (3), calculate the value of the flow Q_2 (where $D = D_2$, $n = n_2$, $S = S_2$ are known); the pipe 02 is considered as a reference pipe with known parameters.

Step 3: Using Eq. (3), calculate the value of the flow Q_3 (where $D = D_3$, $n = n_3$, $S = S_3$ are known).

Step 4: Using the parameters computed above, apply Eq. (16).

Step 5: One estimates the maximum deviation between the the Manning Eq. (3) and the proposed Eq. (16) or (17) using the following formula:

$$\frac{\mathrm{Abs}(\theta_{\mathrm{Manning}} - \theta_{\mathrm{Zeghadnia}})}{\theta_{\mathrm{Manning}}}$$

Step 6: The same steps can be used to evaluate the deviation for Akgiray (2005) or Saâtçi (1990) (as explained above).

For Cases three and four, the same steps can be used as shown above. If the reference pipe was chosen, it will be easy to compute the parameters of the second pipe.

Discussion

After investigation, the authors found that the Saatçi (1990) and Akgiray (2005) approaches are less accurate than the proposed approach as shown in Table 1. In the Saatçi (1990) formula, the maximum deviation for the applicable range of θ (between 0 and 265 degrees) as compared with Eq. (3) is unacceptably high. Similarly, the maximum deviation for the Akgiray (2005) equation in the applicable range of θ is 41.36%, and it is 18.14% for θ between 0° and 301°. On the other side, the proposed approach using Eqs. (16), (17), (20), and (21) in all cases cited above produces an exact solution for all θ values as shown in Table 1.

Table 1. Comparison between Eq. (3) Results and Those of Eq. (16), Saatçi (1990) and Akgiray (2005)

θ_1 and θ_2 (degrees)	Manning Eq. (3) in radian	Proposed Eq. (16) in radian	Error %	Saatçi (1990) Eq. (16) error %	Akgiray (2005) Eq. (9) error %
1	0, 02	0, 02	0, 00	$2{,}69 \times 10^{+4}$	9.56×10^{-3}
2	0, 03	0, 03	0, 00	$1{,}34 \times 10^{+4}$	4.58×10^{-3}
3	0, 05	0, 05	0, 00	$8{,}901 \times 0^{+3}$	3.26×10^{-3}
4	0, 07	0, 07	0, 00	$6{,}65 \times 10^{+3}$	9.93×10^{-3}
5	0, 09	0, 09	0, 00	$5{,}30 \times 10^{+3}$	1.43×10^{-2}
6	0, 10	0, 10	0, 00	$4{,}40 \times 10^{+3}$	2.05×10^{-2}
7	0, 12	0, 12	0, 00	$3{,}76 \times 10^{+3}$	2.91×10^{-2}
8	0, 14	0, 14	0, 00	$3{,}27 \times 10^{+3}$	3.76×10^{-2}
9	0, 16	0, 16	0, 00	$2{,}90 \times 10^{+3}$	4.75×10^{-2}
10	0, 17	0, 17	0, 00	$2{,}60 \times 10^{+3}$	5.86×10^{-2}
20	0, 35	0, 35	0, 00	$1{,}24 \times 10^{+3}$	0.236
35	0, 61	0, 61	0, 00	$6{,}60 \times 10^{+2}$	0.735
45	0, 79	0, 79	0, 00	$4{,}84 \times 10^{+2}$	1.224
90	1, 57	1, 57	0, 00	$1{,}66 \times 10^{+2}$	4.834
100	1, 74	1, 74	0, 00	$1{,}32 \times 10^{+2}$	5.884
120	2, 09	2, 09	0, 00	$7{,}92 \times 10^{+1}$	8.157
135	2, 36	2, 36	0, 00	$4{,}84 \times 10^{+1}$	9.959
145	2, 53	2, 53	0, 00	$3{,}08 \times 10^{+1}$	11.181
190	3, 31	3, 31	0, 00	$2{,}89 \times 10^{+1}$	16.267
200	3, 49	3, 49	0, 00	$3{,}91 \times 10^{+1}$	17.139
235	4, 10	4, 10	0, 00	$6{,}96 \times 10^{+1}$	18.847
245	4, 27	4, 27	0, 00	$7{,}73 \times 10^{+1}$	18.914
290	5, 06	5, 06	0, 00	N/A	17.743
300	5, 23	5, 23	0, 00	N/A	17.985
308	5, 37	5, 37	0, 00	N/A	20.620
335	5, 84	5, 84	0, 00	N/A	32.894
345	6, 02	6, 02	0, 00	N/A	36.612
360	6, 28	6, 28	0, 00	N/A	41.361

Conclusion

This research aims at proposing an analytic solution for the computation of the water surface angle using a known reference pipe's characteristics in the case of pipes arranged in parallel. Based on this approach, the computation of the water surface angle becomes straightforward. The authors have shown that the proposed equations are much better than other approaches. It eliminates the need for calculating the water surface angle, iteratively and produces an exact solution with zero deviation.

Notation

The following symbols are used in this paper:

A = cross section (m^2);
CC1 = subwatershed number 01.
CC2 = subwatershed number 02;
CC3 = equivalent watershed;
M1-N1 = pipe which collect water from subwatershed 01;
M2-N1 = pipe which collect water from subwatershed 02;
N1-N2 = pipe which collect water from the equivalent watershed;
n = channel roughness coefficient (Manning n);
Q = flow rate (m^3/s);
$Q1$ = flow produced in subwatershed 01;
$Q2$ = flow produced in subwatershed 02;
$Q3$ = flow produced in the equivalent watershed;
Rh = hydraulic radius of channel (m);
Rq = ratio of flows;
S = slope of pipe bottom, dimensionless;
V = velocity of flow (m/s); and
θ = water surface angle (rad).

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