Improvement of the Explicit Approximations of the Colebrook Flow Friction Equation Using Wright ω-Function

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Abstract

The Darcy–Weisbach friction factor (f) has been investigated in several studies, and is believed to be affected by material and size of pipe, and the velocity of the flow in pipe as well. The Colebrook–White equation is a complex equation where the computation of the friction factor is not direct, and there is a need for trial-error methods or graphical solutions. Despite profound researches have been made to rectify the equation, there is still no satisfactorily simple yet accurate approach concluded for the most convenient applications in various engineering fields. Therefore this paper is going to improve the results of Dejan Brkić and Pavel Praks using the Wright ω-Function.

Key words: friction factor; explicit solutions; moody diagram; maximum deviation; turbulent flow.

1. Introduction

Colebrook-White Equation (CWE) was introduced by Colebrook and White (Colebrook et al, 1937), which is a combination of von Karman rough pipe equation and Prandtl smooth pipe equation (Karman, 1937), (Prandtl, 1952).

Germano (2020) report that one of the first and notable works addressing this issue was

developed by Prony (1804 apud Rennels and Hudson, 2012), which expressed water pressure drop in internal conduits using two empirical coefficients. Later, Weisbach (1845) proposed the use of a dimensionless group called “friction factor” in a pressure drop equation. Then, Darcy (1857) developed another equation using three empirical coefficients. Although he had not used the dimensionless group proposed by Weisbach (1845), Darcy (1857) had an important role, identifying that pressure drop depends on type and condition of the boundary material. For these reasons, it is traditional to call friction factor as Darcy friction factor or Darcy–Weisbach’s friction factor, although Darcy (1857) had not proposed it, neither Weisbach (1845) had a theorical basis for friction factor meaning.

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The Colebrook –white equation is an empirical formula; it is given by the equation (1) [2]:

$$f = \left[-2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{R_e\sqrt{f}}\right)\right]^{-2} \quad (1)$$

Where:

ε: is the average roughness height (Or the equivalent Nikuradse's sand-grain roughness),

D: is the interne pipe diameter,

F: friction factor,

Re: Reynolds number.

References

1. Dejan Brki´c; and Pavel Praks .Accurate and Efficient Explicit Approximations of the Colebrook Flow Friction Equation Based on the Wright ω-Function" by Dejan Brki´c; and Pavel Praks, Mathematics. 2019, 7, 34; doi:10.3390/math7010034.
2. Colebrook, C.F.; White, C.M. 1937 Experiments with fluid friction in roughened pipes. Proc. Roy. Soc A Mat. 1937, 161, 367–381.
3. Zeghadnia, L.; Robert, J.L., and Achour, B. Explicit Solutions for Turbulent Flow Friction Factor: A Review, Assessment and Approaches Classification. Ain Shams Engineering Journal. 2019, ASEJ-D-18-00215R1 (Accepted).
4. Lawrence, P.W.; Corless, R.M.; Jeffrey, D.J. Algorithm 917: Complex double-precision evaluation of the Wright ω function. ACM Trans. Math. Softw. (TOMS) 2012, 38, 1–17.
5. Wright, E.M. Solution of the equation z • ez = a. Bull. Am. Math Soc. 1959, 65, 89–93.
6. Vatankhah, A.R. Comment on "Gene expression programming analysis of implicit Colebrook–White equation in turbulent flow friction factor calculation". J. Petroleum Sci. Eng. 2014, 124, 402-405.
7. Chandrasekhar S.V.M.; Sharma V.M., 2016, Brownian heat transfer enhancement in the turbulent regime, Facta Universitatis-series Mechanical Engineering. 2016, 14, 169 – 177.
8. Zeghadnia, L., Djemili, L., and Houichi, L.Analytic solution for the computation of flow velocity and water surface angle for drainageand sewer networks: Case of pipes arranged in series.Int. J. Hydrol.Sci. Tech. 2014, 4, 58–67; DOI: 10.1504/IJHST.2014.064413.
9. Zeghadnia, L., Djemili, L., Rezgui, N., and Houichi, L.New equation for the computation of flow velocity in partially filled pipes arranged in parallel." J. Water Sci. Tech. 2014, 70, 160–166; DOI: 10.2166/wst.2014.199.
10. Zeghadnia, L.; Robert, J.L. New Approach for the computation of the water surface angle in partially filled Pipe: Case of Pipes Arranged in Parallel. J. Pipeline Syst. Eng. Pract. 2017, 8, 1-4; DOI: 10.1061/(ASCE)PS.1949-1204.0000272.
11. Karman, V. 1937. The fundamentals of the statistical theory of turbulence. Journal of Aeronautical Sciences 4, 131-138. Prandtl, L. 1952. Essentials of fluid dynamics. Hafner, New York.
12. Rennels, D., & Hudson, H. (2012). Pipe Flow: A Practical and Comprehensive Guide. John Wiley and Sons.
13. Weisbach, J. (1845). Lehrbuch der Ingenieur- und Maschinen-Mechanik. Theoretische Mechanik, Vieweg und Sohn,535.
14. Darcy, H. (1857). Recherches expérimentales relatives au mouvement de l'eau dans les tuyaux. Mallet-Bachelier.
15. Germano Scarabeli Custódio Assunção, Dykenlove Marcelin, João Carlos Von
16. Hohendorff Filho, Denis José Schiozer,Marcelo Souza De Castro. (2020). Friction Factor Equations Accuracy For Single And Two-Phase Flows. Proceedings of the ASME 2020 39th International Conference on Ocean, Offshore and Arctic Engineering OMAE2020, August 3-7, 2020, Virtual, Online.