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RESEARCH ARTICLE

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Nonlinear fractional and singular systems: Nonexistence, existence, uniqueness, and Hölder regularity

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In the present paper, we investigate the following singular quasilinear elliptic system:

$$\begin{cases} (-\Delta)^{s_1}_{p_1} u = \dfrac{1}{u^{\alpha_1} v^{\beta_1}}, & u > 0 \text{ in } \Omega;\ u = 0, \text{ in } \mathbb{R}^N \backslash \Omega, \\ (-\Delta)^{s_2}_{p_2} v = \dfrac{1}{v^{\alpha_2} u^{\beta_2}}, & v > 0 \text{ in } \Omega;\ v = 0, \text{ in } \mathbb{R}^N \backslash \Omega, \end{cases} \tag{S}$$

where $\Omega \subset \mathbb{R}^N$ is an open-bounded domain with smooth boundary, $s_1, s_2 \in (0,1)$, $p_1, p_2 \in (1, +\infty)$, and $\alpha_1, \alpha_2, \beta_1, \beta_2$ are positive constants. We first discuss the nonexistence of positive classical solutions to system (S). Next, constructing suitable ordered pairs of subsolutions and supersolutions, we apply Schauder's fixed-point theorem in the associated conical shell and get the existence of a positive weak solutions pair to (S), turn to be Hölder continuous. Finally, we apply a well-known Krasnosel'skii's argument to establish the uniqueness of such positive pair of solutions.

KEYWORDS

fractional $p-$Laplace equation, nonexistence, quasilinear singular systems, regularity results, Schauder's fixed-point theorem, subhomogeneous problems, subsolutions and supersolutions

MSC CLASSIFICATION

35R11; 35B25; 49J35; 35A16

1 | INTRODUCTION AND STATEMENT OF MAIN RESULTS

Let $\Omega \subset \mathbb{R}^N$ be an open-bounded domain with $C^{1,1}$ boundary, $s_1, s_2 \in (0,1)$, $p_1, p_2 \in (1, +\infty)$, and $\alpha_1, \alpha_2, \beta_1, \beta_2$ are positive constants. In this paper, we are interested in the following nonlocal quasilinear and singular system:

$$\begin{cases} (-\Delta)^{s_1}_{p_1} u = \dfrac{1}{u^{\alpha_1} v^{\beta_1}}, & u > 0 \text{ in } \Omega;\ u = 0, \text{ in } \mathbb{R}^N \backslash \Omega, \\ (-\Delta)^{s_2}_{p_2} v = \dfrac{1}{v^{\alpha_2} u^{\beta_2}}, & v > 0 \text{ in } \Omega;\ v = 0, \text{ in } \mathbb{R}^N \backslash \Omega. \end{cases} \tag{S}$$

