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Chirped localized pulses in a highly nonlinear optical fiber with quintic non-Kerr nonlinearities

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ABSTRACT

We study the existence and propagation properties of chirped localized pulses in a highly nonlinear fiber medium exhibiting self-steepening, self-frequency shift, and quintic non-Kerr nonlinearities. Pulse evolution in such fiber system is governed by a higher-order nonlinear Schrödinger equation incorporating the derivative Kerr and non-Kerr nonlinear terms. We show that bright, dark and kink type solitary waves exist in the presence of all physical processes. A special ansatz is introduced to analyze the existence of solitary waves on a continuous-wave background in the optical fiber medium. It is shown that the obtained localized pulses exhibit a nonlinear chirp which has a quadratic dependence on light intensity. We also find that the magnitude of the associated frequency chirp can be controlled effectively by varying the parameters of self-steepening, self-frequency shift, and derivative non-Kerr nonlinearity effects. The restrictions on the optical fiber parameters are also extracted for the existence of these nonlinearly chirped solitary waves. Results in this study may be useful for experimental realization of shape-preserved pulses in optical fibers and further understanding of their optical transmission properties.

Introduction

Femtosecond light pulses through optical fibers have been widely addressed due to their diverse applications in ultrahigh-bit-rate optical communication systems [1]. It should be noted that their applications also include infrared time-resolved spectroscopy, optical sampling systems, as well as ultrafast physical processes [1,2]. These femtosecond light pulses may thus pave way to generate several important phenomena in field of nonlinear optics. But, experimental and theoretical results have demonstrated that higher-order physical processes of both nonlinear and dispersive nature take place when femtosecond optical pulses propagate inside monomode optical fibers [3]. In particular, when pulses of less than 100 fs duration are injected in the fiber medium, the higher-order nonlinear effects such as the delayed nonlinear response and the self-steepening become significant and should be taken into account [4]. Moreover, the nonlinear refractive index of several optical materials may deviate from the Kerr dependence at higher light intensities, which leads to the appearance of quintic nonlinearity. Experimental results for nonlinear fibers showed that this kind of nonlinearity appears for example in chalcogenide glasses [5], semiconductor-doped glasses [6], and organic polymers [7].

Theoretical descriptions of femtosecond pulse evolution through optical fibers in the presence of these effects support use of the higher-order nonlinear Schrödinger (HNLS) equation to describe the pulse dynamics in such optical media [1]. This model incorporates additional higher-order terms, which describe the influence of different physical phenomena on ultrashort pulse propagation and generation. In general, both the soliton shape and stability properties can change due to such higher-order effects. It is relevant to mention that the NLS-type equations [8] together with other prototypical equations [9–16] have been analyzed from different points of view because of their extensive

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use to describe the soliton dynamics in a wide variety of real physical systems.

Recently, a kind of HNLS equation with quintic non-Kerr terms has been shown to support a new type of Dark-in-the-Bright solitary wave solution also called dipole soliton under some parametric conditions [17]. Due to its physical importance in describing sub-10-fs-pulse propagation in highly nonlinear optical fibers, such equation has been analyzed from different points of view. For instance, Choudhuri and Porsezian [18] have analytically solved this model and obtained results for bright and dark solitary wave solutions with a functional form different from the traditional sech and tanh bright and dark solitons. These authors have also studied the modulational instability (MI) of this HNLS equation in an optical context and presented an analytical expression for MI gain [19]. In addition, Sharma and Goyal [20] have found different types of soliton solutions including bright, dark, double-kink and algebraic solitons of this equation. Moreover, the propagation properties of the dipole solitons have been recently studied in the framework of this model in presence of additional septic non-Kerr nonlinear terms [21,22]. However, for all the studies mentioned above, the solitary wave solutions with nonlinear chirp to this HNLS model in the absence of third-order dispersion have not been recovered yet.

Recently, considerable attention has been focused on the propagation of the nonlinearly chirped soliton pulses through optical fibers [23–26]. The main characteristic of the frequency chirping is amplifying and compressing solitary pulses in nonlinear fibers, and it can have applications in the design of optical fiber compressors, optical fiber amplifiers, and solitary-wave-based communications links [27]. In this setting, physically important chirped solitons has been found in optical media governed by the NLS equation with a source [23] and different HNLS family of equations [2,24,25]. The study of the frequency chirping property has been also extended to chirped Peregrine solitons described by the cubic–quintic NLS equation [26] and chirped dissipative solitons governed by the complex cubic–quintic nonlinear Ginzburg–Landau equation [28].

It is of interest to search for nonlinearly chirped envelope solitons in optical materials exhibiting other types of higher-order nonlinear effects such as the non-Kerr quintic nonlinearities. Recent results indicated that these non-Kerr nonlinear effects are significant in order to adapt to the current progress in high-repetition-rate (beyond ultrashort, even attosecond) optical systems [19,29]. It is interesting to note that the exact balance between different higher-order effects gives rise to a rich variety of nonlinear waveforms that are desired to understand widely different physical phenomena governed by the model equation. In the current study, we analyze the propagation properties of localized pulses with nonlinear chirp under the influence of non-Kerr quintic nonlinearities. In particular, the results presented below demonstrate the possibility of chirped bright, dark and kink solitons existence in optical fibers exhibiting self-steepening, self-frequency shift, and derivative non-Kerr nonlinearities, which may have potential application for the further experiments and research in nonlinear optics. These nonlinearly chirped solitary waves are obtained in the presence of all physical processes. We find that the associated frequency chirp depends quadratically on the intensity of the wave and its amplitude can be controlled efficiently by varying the parameters of derivative Kerr and non-Kerr nonlinearities.

The plan of the current study is drafted as follows. The governing HNLS equation is presented in Section "Evolution equation for the pulse intensity" and the nonlinear equation depicting the dynamics of the pulse intensity in a highly nonlinear optical fiber medium is also derived. Results for analytical chirped bright solitary waves on both zero and on continuous-wave (cw) backgrounds as well as dark and kink solitary wave solutions are presented in Section "Results for chirped solitary waves". We also propose here a special ansatz, whereby it becomes possible to determine the closed form chirped solitary wave solution on cw background of the model. In addition, we present here the frequency chirp related to each of these propagating nonlinear waves and the parametric restrictions for their existence. Finally, Section "Conclusions" concludes the paper.

Evolution equation for the pulse intensity

We describe the transmission of an ultrashort optical pulse inside a highly nonlinear fiber medium when the effect of third-order dispersion is negligible by the HNLS equation [17,18],

$$i\frac{\partial\psi}{\partial z}+\alpha\frac{\partial^2\psi}{\partial t^2}+\gamma\,|\psi|^2\,\psi+i\epsilon\frac{\partial(|\psi|^2\,\psi)}{\partial t}+i\mu\psi\frac{\partial(|\psi|^2)}{\partial t}$$
$$+\upsilon\,|\psi|^4\,\psi+i\delta\frac{\partial(|\psi|^4\,\psi)}{\partial t}+i\sigma\psi\frac{\partial(|\psi|^4)}{\partial t}=0, \tag{1}$$

where ψ is the complex envelope of the electric field, and α, γ, ϵ, and μ are real parameters related to group-velocity dispersion (GVD), self-phase modulation (SPM), self-steepening, and self-frequency shift due to stimulated Raman scattering, respectively. Also, the terms proportional to the coefficients υ, δ, and σ are the quintic non-Kerr terms. It is relevant to mention that the third-order dispersion effect can be neglected for light pulses whose width is of the order of 100 fs or more, having power of the order of 1 W and GVD far away from zero [2,24].

It is interesting to search for analytic solitary wave solutions with nonlinear chirp of the HNLS Eq. (1). To obtain these chirped waveforms, we use an envelope traveling-wave solution of the form [2,23–25],

$$\psi(z,t)=\rho(\xi)\,e^{i[\varphi(\xi)-\kappa z]}. \tag{2}$$

Here the two real quantities φ and ρ representing the phase distribution and field amplitude are functions of the traveling coordinate $\xi=t-uz$, where $u = \mathrm{v}^{-1}$ with v being the group velocity of the light pulse envelope. The accompanying chirp is given by the expression $\delta\omega(z,t)=-(\partial/\partial t)[\varphi(\xi)-\kappa z]=-d\varphi/d\xi$.

Substitution of Eq. (2) into the model (1) and separation of the real and imaginary parts yield the following coupled equations in ρ and φ,

$$\rho\left(\kappa+u\frac{d\varphi}{d\xi}\right)+\alpha\left[\frac{d^2\rho}{d\xi^2}-\rho\left(\frac{d\varphi}{d\xi}\right)^2\right]+\gamma\rho^3+\upsilon\rho^5-\epsilon\rho^3\frac{d\varphi}{d\xi}-\delta\rho^5\frac{d\varphi}{d\xi}=0, \tag{3}$$

and

$$\alpha\left(\rho\frac{d^2\varphi}{d\xi^2}+2\frac{d\varphi}{d\xi}\frac{d\rho}{d\xi}\right)-u\frac{d\rho}{d\xi}+(2\mu+3\epsilon)\,\rho^2\frac{d\rho}{d\xi}+(4\sigma+5\delta)\,\rho^4\frac{d\rho}{d\xi}=0. \tag{4}$$

The multiplication of Eq. (4) by $d\rho/d\xi$ and integration with respect to ξ yields an evolution equation for φ as,

$$\frac{d\varphi}{d\xi}=A\rho^4+B\rho^2+C, \tag{5}$$

where the coefficients A, B and C are expressed in term of the fiber parameters as

$$A=-\frac{4\sigma+5\delta}{6\alpha},\quad B=-\frac{2\mu+3\epsilon}{4\alpha},\quad C=\frac{u}{2\alpha}. \tag{6}$$

Thus the resultant frequency chirp takes the form of $\delta\omega(z,t) = -\left(A\rho^4+B\rho^2+C\right)$, in which A and B are the nonlinear chirp parameters, while C accounts for the constant chirp parameter. This latter expression shows that the associated frequency chirp possesses a nontrivial structure that includes two intensity dependent contributions apart from the linear chirp.

As it is seen from Eq. (6), the parameters of self-steepening and self-frequency shift effects (also called the derivative Kerr nonlinear terms) ϵ and μ and the derivative quintic non-Kerr nonlinear terms σ and δ strongly affect the amplitude and shape of the frequency chirp. A linearity in pulse chirp can be obtained in the absence of the Kerr and non-Kerr nonlinearities. This allows us to conclude that the nonlinearity in the pulse chirp depends crucially on the contribution of the derivative Kerr and non-Kerr nonlinearities in the nonlinear response of waveguiding media.

Placing Relations (5) and (6) into Eq. (3), one obtains the nonlinear differential equation,

$$\frac{d^2\rho}{d\xi^2}+M\rho+N\rho^3+Q\rho^5+R\rho^7+S\rho^9=0, \tag{7}$$

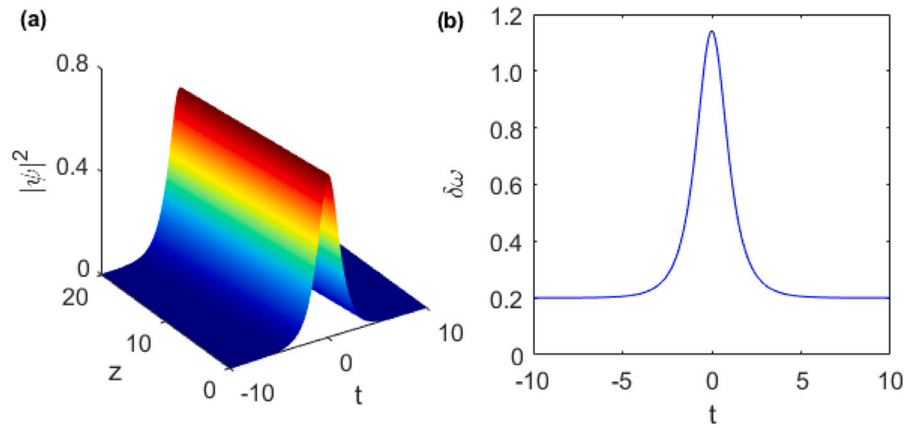

Fig. 1. (a) Evolution of the bright solitary wave (14) and (b) the corresponding chirp profile for $\alpha = 0.25$, $\gamma = -0.1$, $\epsilon = 0.5$, $\upsilon = 0.1$, $\mu = -0.4$, $\delta = 0.6$, $\sigma = -0.3$, and $\kappa = -0.075$.

where

$$M = \frac{u^2 + 4\alpha\kappa}{4\alpha^2}, \quad N = \frac{2\alpha\gamma - u\epsilon}{2\alpha^2}, \quad Q = \frac{16\alpha\upsilon + (2\mu + 3\epsilon)(\epsilon - 2\mu) - 8u\delta}{16\alpha^2},$$
$$R = -\frac{(2\mu + 3\epsilon)(2\sigma + \delta)}{6\alpha^2}, \quad S = \frac{(4\sigma + 5\delta)(\delta - 4\sigma)}{36\alpha^2}. \tag{8}$$

The multiplication of Eq. (7) by $d\rho/d\xi$ and integration with respect to ξ leads to the following nonlinear differential equation,

$$\left(\frac{d\rho}{d\xi}\right)^2 + M\rho^2 + \frac{N}{2}\rho^4 + \frac{Q}{3}\rho^6 + \frac{R}{4}\rho^8 + \frac{S}{5}\rho^{10} + 2\mathcal{E} = 0, \tag{9}$$

with $\mathcal{E}$ being an arbitrary constant of integration that corresponds to the energy of the anharmonic oscillator [18,30].

With the transformation $\rho^2 = F$, the preceding Eq. (9) transforms into the following differential equation,

$$\left(\frac{dF}{d\xi}\right)^2 + 4MF^2 + 2NF^3 + \frac{4Q}{3}F^4 + RF^5 + \frac{4S}{5}F^6 + 8\mathcal{E}F = 0. \tag{10}$$

This nonlinear differential equation describes the dynamics of field intensity in an optical waveguiding medium governed by the HNLS Eq. (1). In the present study, we focus on the analytic localized wave solutions of Eq. (10), when all the physical processes contribute to the nonlinear response of the material. Such analytical solutions will enable us to understand efficiently the transmission properties of nonlinear waves in the optical media.

Results for chirped solitary waves

Having obtained an evolution equation for the pulse intensity [Eq. (10)], we now search for certain solitary waves of the Model (1) through solving that Eq. (10) analytically. Solitary waves on a cw background are also derived by using a special ansatz in the presence of all physical processes.

Bright solitary waves on a zero background

We first analyze the existence of chirped bright solitary waves on a zero background for the model (1). Here we consider the parametric conditions $N = R = 0$, which implies that $u = 2\alpha\gamma/\epsilon$ and $(2\mu + 3\epsilon)(2\sigma + \delta) = 0$. We obtain a bright-type solitary wave of Eq. (10) with zero energy ($\mathcal{E} = 0$) in the following form,

$$F(\xi) = \left[\frac{P}{\cosh^2(\eta\xi) + D}\right]^{1/2}, \tag{11}$$

where

$$P = -\frac{3M(2D+1)}{Q}, \quad \eta = 2\sqrt{-M}, \tag{12}$$

$$2D + 1 = \left[1 - \frac{36MS}{5Q^2}\right]^{-1/2}, \tag{13}$$

provided that $M < 0$, $Q > 0$ and $S < \left|\frac{5Q^2}{36M}\right|$.

Hence, we obtain a chirped localized solution for the HNLS equation (1) as,

$$\psi(z,t) = \left[\frac{P}{\cosh^2(\eta\xi) + D}\right]^{1/4} e^{i[\varphi(\xi) - \kappa z]}. \tag{14}$$

The associated frequency chirp takes the form,

$$\delta\omega(z,t) = -A\left(\frac{P}{\cosh^2(\eta\xi) + D}\right) - B\left(\frac{P}{\cosh^2(\eta\xi) + D}\right)^{1/2} - C. \tag{15}$$

Shown in Figs. 1(a) and 1(b) are the intensity and chirp (for $z = 0$) profiles of solitary wave solution (14), respectively. Parameter values we used in the figure are $\alpha = 0.25$, $\gamma = -0.1$, $\epsilon = 0.5$, $\upsilon = 0.1$, $\mu = -0.4$, $\delta = 0.6$, $\sigma = -0.3$, and $\kappa = -0.075$. From Fig. 1(a), we see that the bright solitary wave profile appears on a zero background. We also observe that the pulse profile remains unchanged during evolution. We have found that the numerical results obtained by solving the model (1) by means of the split-step Fourier method and using the analytic solitary wave solution as an initial condition, also confirm this behavior. In addition, we see that the chirp $\delta\omega$ is also localized, as the solitary wave is, but on a nonzero background [Fig. 1(b)].

If we change the values of derivative non-Kerr nonlinear coefficients as $\delta = -0.6$ and $\sigma = 0.3$, we see that the solitary wave keeps the same characteristic intensity profile [Fig. 2(a)] while the chirp presents a shape like a "M"which has two symmetrical humps and one valley in the middle of the chirp profile [Fig. 2(b)]. Therefore the parameters of derivative non-Kerr nonlinearities have a strong influence on the amplitude of the pulse chirp.

Choosing the values of the self-frequency shift and self-steepening coefficients as $\mu = -0.45$ and $\epsilon = 0.1$, we observe that the chirp profile takes the shape of a "W" which contains one hump and two valleys on the hump's two sides [Fig. 3(b)], while the solitary pulse keeps its bright localized structure on a zero background [Fig. 3(a)]. Thus, in order to control the frequency chirp associated with the obtained solitary wave, one should vary the parameters of self-steepening, self-frequency shift, and derivative non-Kerr nonlinearities.

Kink solitary waves

We now find the kink-type solitary wave solutions with nonlinear chirp for Eq. (1) under the same parametric conditions as in the previous solution (11). We should note that these solitary waves are of particular significance in optical fibers [3].

We have found that Eq. (10) for zero energy ($\mathcal{E} = 0$) satisfies a kink solitary wave solution of the form,

$$F(\xi) = \Lambda\left[1 + \frac{\tanh(w\xi)}{1 + \operatorname{sech}(w\xi)}\right]^{1/2}, \tag{16}$$

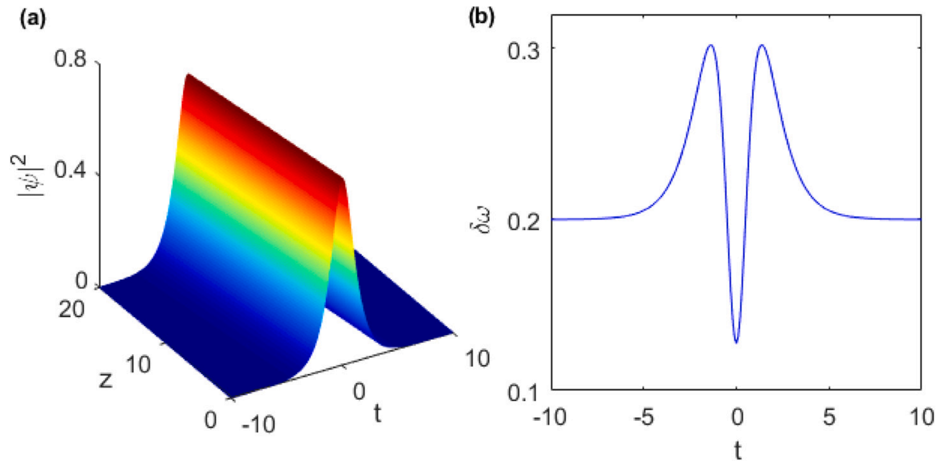

Fig. 2. (a) Evolution of the bright solitary wave (14) and (b) the corresponding chirp profile for $\delta = -0.6$ and $\sigma = 0.3$. The other parameters are the same as given in Fig. 1.

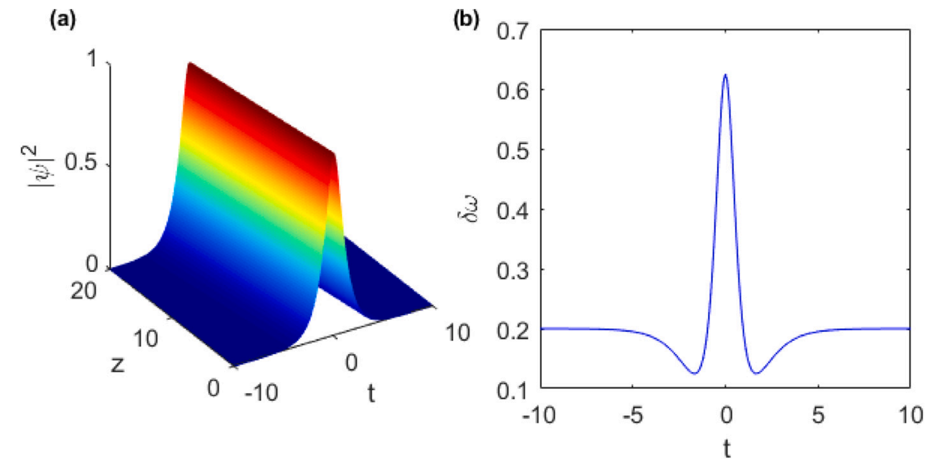

Fig. 3. (a) Evolution of the bright solitary wave (14) and (b) the corresponding chirp profile for $\epsilon = 0.1$ and $\mu = -0.45$. The other parameters are the same as given in Fig. 1.

where

$$w = 4\sqrt{-M}, \tag{17}$$

$$\Lambda = \left(\frac{5M}{4S}\right)^{1/4}, \tag{18}$$

under the parametric condition,

$$Q = -\sqrt{\frac{36MS}{5}}, \tag{19}$$

with $M < 0$ and $S < 0$.

Hence, the chirped kink solitary wave solution of Eq. (1) takes the form,

$$\psi(z,t) = \sqrt{\Lambda}\left[1 + \frac{\tanh(w\xi)}{1 + \mathrm{sech}(w\xi)}\right]^{1/4} e^{i[\varphi(\xi) - \kappa z]}. \tag{20}$$

In this case, the chirping reads as,

$$\delta\omega(z,t) = -A\Lambda^2\left(1 + \frac{\tanh(w\xi)}{1 + \mathrm{sech}(w\xi)}\right) - B\Lambda\left(1 + \frac{\tanh(w\xi)}{1 + \mathrm{sech}(w\xi)}\right)^{1/2} - C. \tag{21}$$

It is worthy to mention that the existence of the preceding kink solitary wave is based on the constraint condition (19) which describes the balance among GVD, quintic nonlinearity, self-frequency shift, and derivative quintic non-Kerr nonlinearities. Unlike the conventional dark solitary wave in Kerr media, the amplitude of the kink solution (20) may approach nonzero when the variable ξ approaches infinity ($|\xi| \to \infty$).

The intensity and chirp (for $z = 0$) profiles of the kink solitary wave (20) are depicted in Figs. 4(a) and 4(b), respectively. Here, we have taken the parameter values as $\alpha = 1$, $\gamma = -0.1$, $\epsilon = 2$, $\upsilon = 0.1$, $\mu = -3$, $\delta = 0.1$, $\sigma = 0.2$, and $\kappa = -0.065$. From Fig. 4(b), one can see that the chirp saturates at two different finite values as $t \to \pm\infty$.

Chirped solitary waves on a cw background

A challenging problem is the search for different types of solitary waves in the presence of all physical processes involved in the propagation Eq. (1). No doubt, the finding of analytical localized wave solutions under the influences of various higher-order effects will help one to understand the transmission properties of nonlinear waves propagating through the nonlinear fiber medium.

To find the analytic solitary wave solution of Eq. (10) in the general case when the coefficients M, N, Q, R, S, and $\mathcal{E}$ have nonzero values, we introduce a special *ansatz* as

$$F(\xi) = \lambda + \Gamma \mathrm{sech}^{1/2}(q\xi), \tag{22}$$

where λ, Γ and q are real parameters to be determined.

Here, in the ansatz (22), the parameter λ decides the strength of the background in which the solitary pulse propagates in the optical fiber, and Γ determines the pulse amplitude while q denotes its width.

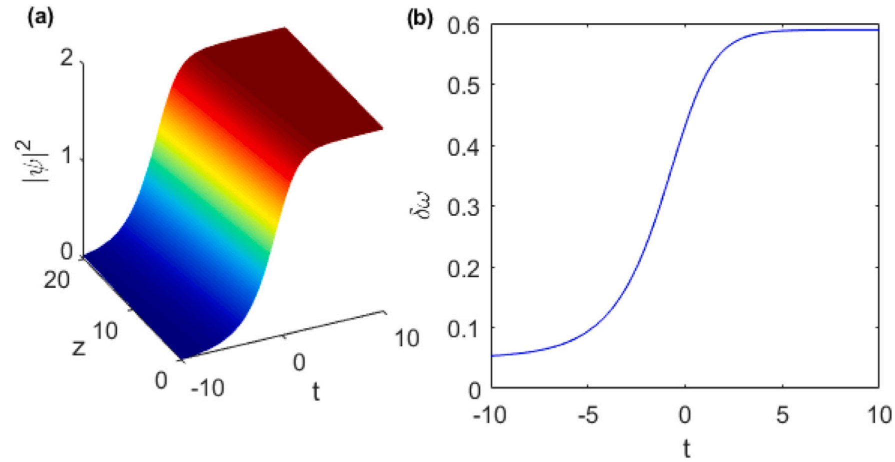

Fig. 4. (a) Evolution of the kink solitary wave (20) and (b) the corresponding chirp profile for $\alpha = 1$, $\gamma = -0.1$, $\epsilon = 2$, $\upsilon = 0.1$, $\mu = -3$, $\delta = 0.1$, $\sigma = 0.2$, and $\kappa = -0.065$.

Inserting this ansatz into Eq. (10) and setting the coefficients of $\mathrm{sech}^n(q\xi)$ (with $n = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3$) to zero, we get the following set of algebraic equations:

$$\lambda\left[4M\lambda + 2N\lambda^2 + \frac{4Q}{3}\lambda^3 + R\lambda^4 + \frac{4S}{5}\lambda^5 + 8\mathcal{E}\right] = 0, \tag{23}$$

$$\Gamma\left[8M\lambda + 6N\lambda^2 + \frac{16Q}{3}\lambda^3 + 5R\lambda^4 + \frac{24S}{5}\lambda^5 + 8\mathcal{E}\right] = 0, \tag{24}$$

$$\Gamma^2\left[\frac{q^2}{4} + 4M + 6N\lambda + 8Q\lambda^2 + 10R\lambda^3 + 12S\lambda^4\right] = 0, \tag{25}$$

$$\Gamma^3\left[2N + \frac{16Q}{3}\lambda + 10R\lambda^2 + 16S\lambda^3\right] = 0, \tag{26}$$

$$\Gamma^4\left[\frac{4Q}{3} + 5R\lambda + 12S\lambda^2\right] = 0, \tag{27}$$

$$\Gamma^5\left[R + \frac{24S}{5}\lambda\right] = 0, \tag{28}$$

$$\Gamma^2\left[\frac{q^2}{4} - \frac{4S}{5}\Gamma^4\right] = 0. \tag{29}$$

These equations can be solved to obtain the solitary wave parameters λ, Γ and q as,

$$\lambda = -\frac{5R}{24S}, \tag{30}$$

$$\Gamma = \pm\lambda, \tag{31}$$

$$q^2 = \frac{16S}{5}\lambda^4, \tag{32}$$

together with the energy value,

$$\mathcal{E} = -\frac{2S}{5}\lambda^5, \tag{33}$$

and the parameters,

$$M = \frac{14S\lambda^4}{5}, \quad N = -8S\lambda^3, \quad Q = 9S\lambda^2. \tag{34}$$

Equating the parameters in (8) and (34), we get the expressions of the inverse group velocity u as,

$$u = \frac{2\alpha\left(\gamma + 8\alpha S\lambda^3\right)}{\epsilon}, \tag{35}$$

and the wave number κ,

$$\kappa = \frac{56S\alpha^2\lambda^4 - 5u^2}{20\alpha}, \tag{36}$$

together with the value of parameter υ,

$$\upsilon = \frac{8u\delta - (2\mu + 3\epsilon)(\epsilon - 2\mu) + 144S\alpha^2\lambda^2}{16\alpha}. \tag{37}$$

Hence, we get the following analytic nonlinearly chirped solitary wave solution on a cw background for Eq. (1),

$$\psi(z,t) = \sqrt{\lambda \pm \lambda\,\mathrm{sech}^{1/2}\left[\sqrt{\frac{16S\lambda^4}{5}}\,(t-uz)\right]}\, e^{i[\varphi(\xi) - \kappa z]}. \tag{38}$$

The corresponding chirp is given by,

$$\delta\omega(z,t) = -A\left(\lambda \pm \lambda\,\mathrm{sech}^{1/2}\left[\sqrt{\frac{16S\lambda^4}{5}}\,(t-uz)\right]\right)^2 - B\left(\lambda \pm \lambda\,\mathrm{sech}^{1/2}\left[\sqrt{\frac{16S\lambda^4}{5}}\,(t-uz)\right]\right) - C. \tag{39}$$

From the relations (30)–(32), we see that the solitary wave parameters λ, Γ and q are uniquely dependent on the parameters R and S. This implies that the parameters related to the derivative Kerr and non-Kerr nonlinear terms are thus essential in obtaining the nonlinearly chirped solitary wave solution (38) for Eq. (1). It is apparent from Eq. (32) that the reality of pulse width q requires $S > 0$, thus implying $(4\sigma + 5\delta)(\delta - 4\sigma) > 0$.

Additionally, the expression (38) demonstrates the existence of two different types of solitary waves for the model (1). A first solution with the top sign describes a bright solitary pulse on a cw background and a second solution with bottom sign depicts a dark-type solitary wave. Figs. 5(a) and 5(b) show the intensity and chirp (for $z = 0$) profiles of the bright solitary wave on a cw background [Eq. (38) with the top sign] for the material parameters $\alpha = 0.25$, $\gamma = -0.485$, $\epsilon = 0.105$, $\upsilon = 0.549$, $\mu = 0.05$, $\delta = 0.5$, $\sigma = 0.1$, and $\kappa = 0.529$. The results of the nonlinear evolution of the dark solitary wave [Eq. (38) with the bottom sign] and its corresponding chirp profile (for $z = 0$) are shown in Figs. 6(a) and 6(b) respectively, by using the same material parameters as those in Fig. 5. We should note that these nonlinearly chirped solitary waves characteristically exist due to a balance among GVD, cubic and quintic nonlinearities, self-steepening, self-frequency shift, and derivative non-Kerr nonlinearities.

For the completeness of the investigation, we now analyze the stability of the obtained chirped solitary waves with respect to the finite initial perturbations. Note that only stable optical solitary waves can be observed experimentally. It is therefore crucial to analyze the stability of the optical localized waves with respect to the finite initial perturbations. These may be random noises, amplitude perturbation, and the slight violation of the parametric conditions [17]. It is relevant to mention that significant results have been obtained with previous theoretical studies concerning stability properties of solitary pulses in systems exhibiting cubic nonlinearity [31]. In addition, it was found that competing nonlinearities occurring in cubic–quintic media can stabilize soliton solutions [32]. In what follows, we analyze the stability

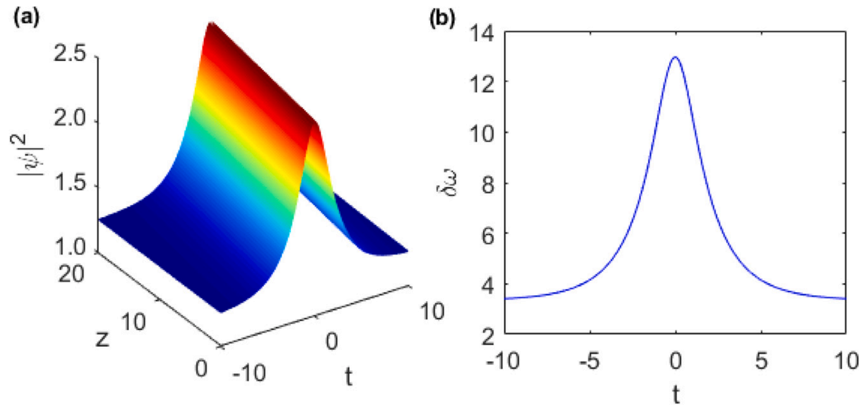

Fig. 5. (a) Evolution of the dark solitary wave (38) and (b) the corresponding chirp profile for $\alpha = 0.25$, $\gamma = -0.485$, $\epsilon = 0.105$, $\upsilon = 0.549$, $\mu = 0.05$, $\delta = 0.5$, $\sigma = 0.1$, and $\kappa = 0.529$.

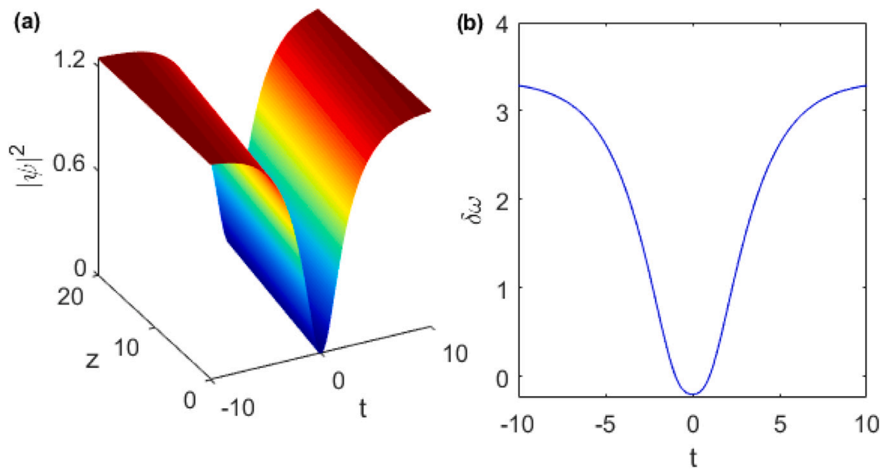

Fig. 6. (a) Evolution of the bright solitary wave on a cw background (38) and (b) the corresponding chirp profile for $\alpha = 0.25$, $\gamma = -0.485$, $\epsilon = 0.105$, $\upsilon = 0.549$, $\mu = 0.05$, $\delta = 0.5$, $\sigma = 0.1$, and $\kappa = 0.529$.

of the obtained chirped solitary wave solutions with respect to the finite perturbations by employing numerical simulations. Here, we performed a direct numerical simulation of Eq. (1) using the standard split-step Fourier method [33], to test the stability of solutions (14), (20) and (38) with initial white noise, as compared to Figs. 1(a), 4(a), 5(a) and 6(a) respectively. As usual, we put the noise onto the initial profile, then the perturbed pulse reads [34]: $\psi_{\text{pert}} = \psi(t,0)[1 + 0.1\,\text{random}(t)]$. The numerical results of bright pulse on a zero background, kink pulse, bright pulse on a cw background, and dark pulse solutions under the perturbation of 10% white noise are displayed in Figs. 7(a), 7(b), 7(c) and 7(d) respectively. From Fig. 7, we can see that under finite initial perturbations of the additive white noise, the solitary waves can still propagate very stably for a rather long distance, with profiles agreeing very well with the analytical solutions. Therefore, we can conclude that the solutions we obtained are stable and should be observable in optical fibers with quintic non-Kerr nonlinearities.

Before we leave this section, we would like to compare the obtained chirped bright solitary waves (14) and (38) with the bright solitons that are experimentally observed in single-mode silica-glass fibers by Mollenauer et al. [35]. Different from the bright soliton with a sech-type wave form obtained within the framework of the cubic NLS equation in the anomalous dispersion regime [36,37], the bright solitary waves presented here possess a nonlinear chirp which depends on the intensity of the pulse. This interesting frequency chirping property may find various practical applications in achieving effective pulse compression or amplification. Noting here that, the formation of the NLS bright soliton in a pure Kerr medium occurs in conditions where only the two basic effects, which are the self-phase modulation nonlinearity and anomalous group velocity dispersion may be balanced. This differs from those determined in the present study where their existence in the fiber medium requires a balance among higher-order effects of different nature.

Conclusions

In conclusion, we have analyzed the existence and propagation properties of chirped solitary pulses in a highly nonlinear optical fiber exhibiting quintic non-Kerr nonlinearities. The transmission of femtosecond light pulses inside such system is described by the HNLS equation combining the derivative Kerr with non-Kerr nonlinear terms. We have found that the evolution equation for the pulse intensity obeys a first order nonlinear ordinary differential equation with at most a sixth-degree nonlinear term. Various types of chirped localized waves including chirped bright, dark and kink solitary wave solutions have been identified in the presence of all physical processes. In addition, nonlinearly chirped bright solitary waves on a continuous-wave background have been obtained by using a special ansatz. The formation conditions of the chirped localized structures have been also presented. It is demonstrated that the frequency chirp accompanying these localized structures has a quadratic dependence on light intensity and its amplitude depends upon the parameters of self-steepening, self-frequency shift, and derivative non-Kerr nonlinearities. These results may be useful for a further understanding of physical phenomena and dynamical processes that arise in highly nonlinear optical fibers described by the present model. Due to the interesting frequency chirp property, the findings of the current paper paves way to potential

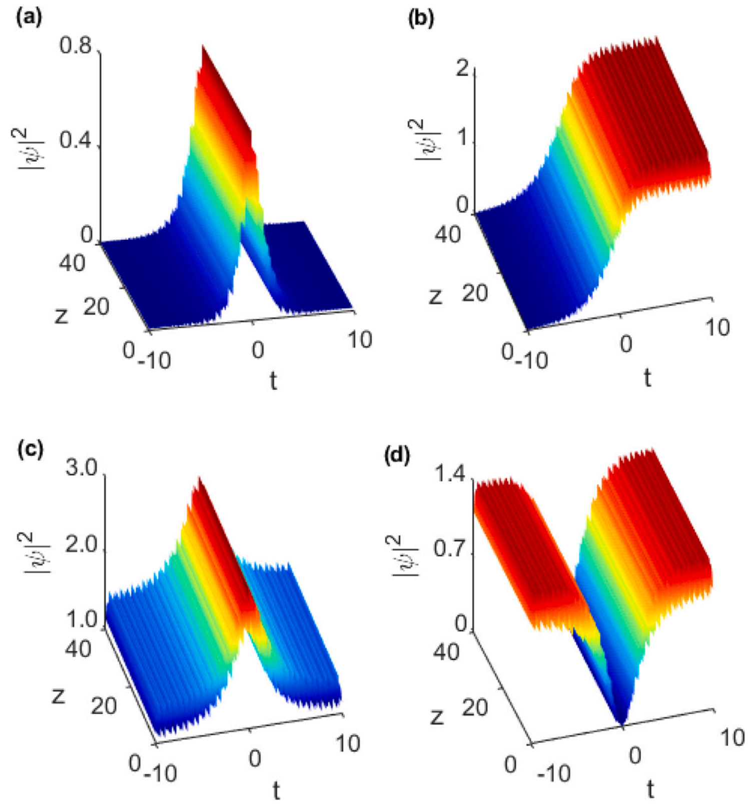

Fig. 7. Numerical evolution of the intensity $|\psi|^2$ for (a) the bright solitary wave solution (14), (b) kink solitary wave solution (20), bright solitary wave solution on a cw background (38), and (c) dark solitary wave solution (38) under the perturbation of white noise whose maximal value is 0.01. The parameters are the same as in Figs. 1(a), 4(a), 5(a) and 6(a) respectively.

applications in optical communications, optical fiber compressors and optical fiber amplifiers.

CRediT authorship contribution statement

Faissal Mansouri: Writing – original draft. **Sassi Aouadi:** Writing – original draft. **Houria Triki:** Writing – review & editing. **Yunzhou Sun:** Investigation. **Yakup Yıldırım:** Writing – review & editing. **Anjan Biswas:** Methodology. **Hashim M. Alshehri:** Methodology. **Qin Zhou:** Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

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