NON-LOCAL SCHRÖDINGER-MAXWELL SYSTEM INVOLVING FRACTIONAL P-LAPLACIAN WITH SINGULAR NONLINEARITY

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Abstract. We consider the non-local system with singular non-linearity and singular weights:

$$\begin{cases} (-\Delta)_p^s u + v\, u^{p-1} = h(x)u^{-\alpha}, & u > 0 \quad \text{in } \Omega; \quad u = 0, \quad \text{in } \mathbb{R}^N \setminus \Omega, \\ (-\Delta)_p^s v = u^p, & v > 0 \quad \text{in } \Omega; \quad v = 0, \quad \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $0 < s < 1$, $p > 1$, $\alpha > 0$, and $\Omega \subset \mathbb{R}^N$, with $N > sp$, is an open bounded domain with $C^{1,1}$ boundary $\partial\Omega$. The function $h : \Omega \to \mathbb{R}^+$ exhibits growth of negative powers of the distance function $d(x) := \text{dist}(x, \partial\Omega)$ near the boundary, that is, $h(x) \sim d^{-\beta}(x)$ for some $\beta \geq 0$, when x is close to the boundary $\partial\Omega$. For $\beta < sp$, we discuss the existence of a positive weak solution $(u, v) \in W^{s,p}_{\text{loc}}(\Omega) \times W^{s,p}_{\text{loc}}(\Omega)$ using the classical method of regularization and the fixed point theorem together. Indeed, we found some essential uniform a priori estimates for the approximating sequence before proceeding to the limits. Moreover, we address the uniqueness of finite energy solutions, i.e., $(u, v) \in W^{s,p}_0(\Omega) \times W^{s,p}_0(\Omega)$, and demonstrate that this solution pair is a saddle point of a suitable functional when $\alpha < 1$. We also provide the boundary behavior of the weak solutions in terms of the distance function. Finally, we establish the non-existence of a weak solution for the case where $\beta \geq sp$.

Keywords: Schrödinger-Maxwell system; Fractional p-Laplace operator; Singular nonlinearity; Existence and non-existence results.

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