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Global convergence of a modified hybrid **DY** and **HS** conjugate gradient method for non convexe optimization

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Résumé :The conjugate gradient method is a useful and powerful approach for solving large-scale minimization problems. Dai and Yuan developed a conjugate gradient method, which has good numerical performance and global convergence result under line searches such as Wolfe and strong Wolfe line search .Recently, we propose a modification of the Dai–Yuan conjugate gradient algorithm, which produces a descent search direction at every iteration and converges globally provided that the line search satisfies the weak Wolfe conditions.

0.1 Main results

Consider an unconstrained minimization problem

$$\min f(x), \quad x \in \mathbb{R}^n, \tag{1}$$

where $\mathbb{R}^n$ denotes an n-dimensional Euclidean space and
$f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is a continuously differentiable function
Generally, a line search method takes the form
$x_{k+1} = x_k + \alpha_k d_k, \quad k = 0, 1, 2, ...$

where d_k is a descent direction of $f(x)$ at x_k and α_k is a step size. For convenience, we denote $\nabla f(x_k)$ by g_k,
The search direction d_k is generally required to satisfy
$g_k d_k < 0$,

which guarantees that d_k is a descent direction of $f(x)$ at x_k [1].
In order to guarantee the global convergence, we sometimes require d_k to satisfy a sufficient descent condition
$g_k^T d_k \leq -c \|g_k\|^2$,

where $c > 0$ is a constant.

In line search methods, the well-known conjugate gradient method has the form (2) in which-

$$d_k = \begin{cases} -g_k, & \text{if} \quad k = 0; \\ -g_k + \beta_k d_{k-1}, & \text{if} \quad k \geq 1, \end{cases}$$

where $\beta_k^{FR} = \frac{\|g_k\|^2}{\|g_{k-1}\|^2}$, $\beta_k^{PRP} = \frac{g_k^T(g_k - g_{k-1})}{\|g_{k-1}\|^2}$, $\beta_k^{HS} = \frac{g_k^T(g_k - g_{k-1})}{d_{k-1}^T(g_k - g_{k-1})}$, $\beta_k^{DY} = \frac{\|g_k\|^2}{d_{k-1}^T y_{k-1}}$,

One often requires the inexact line search such as theWolfe conditions or the strongWolfe conditions. TheWolfe line search is to find $\alpha_k$such that

$$f(x_k + \alpha d_k) \leq f(x_k) + \rho\alpha\nabla^T f(x_k) d_k$$
$$\nabla^T f(x_k + \alpha d_k) d_k \geq \sigma \nabla^T f(x_k) d_k \qquad \text{with} \quad 0 < \rho < \sigma < 1,$$

Al-Baali [10] has proved the global convergence of the **FR** method for nonconvex functions with the strongWolfe line search if the parameter $\sigma < \frac{1}{2}$. The **PRP** method with exact line search may cycle without approaching any stationary point, see Powell's counter-example [11]. Although one would be satisfied with its global convergence properties, the **FR** method sometimes performs much worse than the **PRP** method in real computations. A similar case happen to the **DY** method and the **HS** method. To combine the good numerical performance of the **PRP** and **HS** methods and the nice global convergence properties of the **FR** and **DY** methods, Touati-Ahmed and Storey [12] proposed a hybrid **PRP–FR** method which we call the **H1** method, that is,

$$\beta_k^{H_1} = \max\left\{0, \min\left\{\beta_k^{PRP}, \beta_k^{FR}\right\}\right\}$$

Gilbert and Nocedal [13] extended this result to the case that

$$\beta_k = \max\left\{-\beta_k^{FR}, \min\left\{\beta_k^{PRP}, \beta_k^{FR}\right\}\right\}$$

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