Circle-Criterion Based Nonlinear Observer Design for Sensorless Induction Motor Control

W. Bourbia
Laboratoire Systèmes Electromécaniques, Université Badji Mokhtar Annaba, B.P.12 23000 Annaba

F. Berrezzek
Laboratoire Systèmes Electromécaniques, Université Badji Mokhtar Annaba, B.P.12 23000 Annaba

B. Bensaker
Laboratoire Systèmes Electromécaniques, Université Badji Mokhtar Annaba, B.P.12 23000 Annaba

Abstract--**This paper deals with the design of a nonlinear observer for sensorless induction motor control. It is well-known that induction motor is one of the widely used machines in industrial applications. However, induction motor is also known as a complex nonlinear system in which time-varying parameters entail additional difficulties for the induction machine system control. Based upon the circle criterion approach a nonlinear observer is designed to estimate pertinent but unmeasurable state variables of the machine for sensorless control purpose. The observer gain matrices are computed as a solution of linear matrix inequalities (LMI) that ensure the stability conditions of the state observer error dynamics in the sense of Lyapunov concepts. Measured and estimated state variables can be used to improve the effectiveness of a state feedback control of the considered induction motor system. The simulation results are presented to illustrate the validity of the proposed approach for nonlinear observer design.**

Index Terms—**Nonlinear observer, Circle criterion, LMI-based, Lyapunov function, Induction motor.**

1. INTRODUCTION

Induction motor is one of the most widely used machines in industrial applications. This is due to its high reliability, relatively low cost, and modest maintenance requirements. However, induction motor is also known as a complex nonlinear system, in which time-varying parameters entails additional difficulties for machine control, conditions monitoring and fault diagnostic purposes [1]. Due to technical and/or economical constraints only a few state variables of the machine are available for on-line measurement in industrial applications. In order to perform advanced control techniques there is a great need of a reliable and accurate estimation of key unmeasurable state variables of the machine.

It is well known from control theory that a state estimator, called also state observer, is a dynamic system that is driven by the input-output of the considered system, estimates asymptotically its unmeasurable state variables. It uses an adaptive mechanism involving as input, the error between the measured and estimated output values of the system. It is a "software sensor" that plays an important role in the estimation of the unmeasurable (internal) state variables that are essential not only in sensorless control techniques and conditions monitoring but also in fault diagnosis and predictive system maintenance.

A control literature review shows that nonlinear observer design approaches can be roughly divided into three classes. The first class of approaches attempts to eliminate the system nonlinearities by a technique of linearization or a nonlinear state transformation to linearize the original system [2], [3]. Its drawback is a set of extremely restrictive conditions that can hardly be met by a physical system. In this context of linearization approach, one could mention the extended Luenberger observer and extended Kalman filter. The second class of approaches is the high-gain observer-based approach which attempts to dominate the system nonlinearities by a unique high gain output correction term [4]. High-gain observers are robust state estimator and disturbances attenuator. However, their drawbacks are the block triangular structures, the destabilizing effect of the peaking phenomenon (large oscillation in the transient response) and sensitivity against measurement noises.

The third class of approaches to design nonlinear observers exploits directly the system nonlinearity. Lipschitz and sector properties are the main nonlinearity properties that are exploited [5], [6], [7]. This latest class of approaches to design nonlinear observer for nonlinear systems has been recently developed. Now, it has reached the maturity to be exploited in the machinery application to benefit from its advantages.

In the machinery community several approaches from control theory have been applied for the design of nonlinear observer to control electric machines. The main ones are linearization approaches, high-gain observers, Lyapunov-based approach, geometric algorithms [8], sliding-modes (variable structure) design procedures [9], and algebraic techniques [10]. Sliding mode observer is a particular type of variable structure observer. It is designed to force the system state estimation error to lie within a neighbourhood of a switching function. It incorporates robustness property against a range of system uncertainties and disturbances.

Our main goal here is to show that recently developed techniques for nonlinear observer design based upon the sector property, can be exploited for electric machine sensorless control and conditions monitoring. In this paper, we focus our attention on the application of the so-called circle-criterion approach to design a nonlinear observer for induction motor sensorless control. The advantage of the circle criterion approach is that it directly handles the nonlinearities of the system with less difficulty and less restriction conditions. The circle criterion approach to design nonlinear observer is a new line of research introduced for continuous-time systems by [5]. An extension to multi-variable discrete-time case is given in [7]. In the following we recall the ingredient of the circle criterion nonlinear observer design approach.

2. CIRCLE-CRITERION BASED NONLINEAR OBSERVER DESIGN

In contrast of the linearization-based and high-gain approaches which attempt to eliminate the system nonlinearities using a nonlinear state transformation or to dominate them by a high gain term of correction, circle-criterion approach to nonlinear observer design exploits the type of system nonlinearities. In its basic form, introduced by Arcak and Kokotovic [5], the approach is applicable to a class of systems that can be decomposed in linear and nonlinear parts with a condition that the nonlinearities satisfy the sector property.

A. Basic sector property

A memoryless function $f(z,t):[0\ +\infty[\times R^p \to R^p$ is said to belong to the sector $[0\ +\infty[$ if $zf(z,t)\geq 0$. Let v_1 and v_2 two real positive numbers, by setting $z=v_1-v_2$ and $f(z,t)=[f(v_1,t)-f(v_2,t)]$, the above sector property is equivalent to:

$$(v_1-v_2)[f(v_1,t)-f(v_2,t)]\geq 0\ \ \forall v_1,v_2\in R^+ \quad (1)$$

Relation (1) states that the function $f(z,t)$ is a nondecreasing function. On the other hand if $f(z,t)$ is a continuously differentiable function the above relation is equivalent to [5], [6]:

$$\frac{d}{dz}f(z,t)\geq 0 \quad \forall z\in R \quad (2)$$

If the function $f(z,t)$ does not satisfy the positivity condition (2) we introduce a function $g(z,t)$ such that:

$$g(z,t)=f(z,t)+\rho z, \quad \rho\succ\left\|\frac{d}{dz}f(z,t)\right\|,\forall z\in R \quad (3)$$

And

$$\frac{d}{dz}g(z,t)=\frac{d}{dz}f(z,t)+\rho\geq 0 \quad \forall z\in R \quad (4)$$

In the multivariable case the sector property can be written as: $z^T f(z,t)\geq 0$. Where z and $f(z,t)$ are respectively vectors of an appropriate dimension.

B. Nonlinear Observer Design

The circle criterion based nonlinear observer design can be performed for a class of nonlinear system that the model can be decomposed into linear part and nonlinear part as the following [5], [6], [7]:

$$\dot{x}(t)=Ax(t)+\phi[u(t),y(t)]+Gf[H.x(t)] \quad (5)$$

$$y(t)=Cx(t) \quad (6)$$

Where A, C and G are known constant matrices with appropriate dimensions. The pair (A,C) is assumed to be observable. The term $\phi[y(t),u(t)]$ is an arbitrary real-valued vector that depends only on the system measured control inputs $u(t)$ and outputs $y(t)$. The nonlinear part of the system is included in the term $f[H.x(t)]$ which is a time-varying vector function verifying the sector property. In the following we recall the main theorem and conditions that are used in this work to study the feasibility of nonlinear observer design for induction motor sensorless control with respect of circle criterion or sector property. A detailed proof of the theorem is presented.

Theorem [5], [6]**:** Consider a nonlinear system of the form (5)-(6) with the nonlinear part satisfying the circle criterion relations (1)-(4). If there exist a symmetric and positive definite matrix $P\in R^{nxn}$ and a set of row vectors $K\in R^p$ such that the following linear matrix inequalities (LMI) hold:

$$(A-LC)^T P+P(A-LC)+Q\leq 0 \quad (7)$$

$$PG+(H-KC)^T=0 \quad (8)$$

With $Q=\varepsilon I_n$ as a defined positive known matrix, I_n is an n-th order unity matrix and ε is a small positive real number.
Then a nonlinear observer can be designed as:

$$\begin{aligned}\dot{\hat{x}}(t)=A\hat{x}(t)+\phi[u(t),y(t)]+L[y(t)-\hat{y}(t)]+\\ +G\ f[H\hat{x}(t)+K(y(t)-\hat{y}(t))]\end{aligned} \quad (9)$$

$$\hat{y}(t)=C\hat{x}(t) \quad (10)$$

And $\lim_{t\to\infty} e(t) = x(t) - \hat{x}(t) = 0$, where $\hat{x}(t)$ is the estimate of the state vector $x(t)$ of the nonlinear system.

The nonlinear observer design refers to the selection of the gain matrices L and K satisfying the LMI conditions (7)-(8). One can see that the structure of the nonlinear observer is composed of a linear part that is similar to linear Luenberger observer and a nonlinear part that is an additional term that represents the time-varying nonlinearities satisfying the sector property. The circle criterion based nonlinear observer design takes advantage of the sector property by introducing a nonlinear term, in the structure of the observer. In the light of the summary proof presented in [5] and [6] we present in the following a detailed proof of the theorem.

Proof. (See [5], [6]).

The state estimation error is given as: $e(t) = x(t) - \hat{x}(t)$, where $\hat{x}(t)$ is the estimate of the state vector $x(t)$ of the nonlinear system (5)-(6). The dynamics of the state estimation error are then:

$$\begin{aligned}\dot{e}(t) &= (A - LC)e(t) + \\ &+ G[f(H.x(t)) - f(H.\hat{x}(t) + K(y(t) - \hat{y}(t))] \end{aligned} \quad (11)$$

Let $v_1 = H.x(t)$ and $v_2 = H.\hat{x}(t) + K(y(t) - \hat{y}(t))$. By setting $z = v_1 - v_2 = (H - KC)e(t)$ the term between brackets in (11) can be seen as a function of the variable z and then: $[f(v_1) - f(v_2)] = f(z,t)$.

The expression $(v_1 - v_2)[f(v_1) - f(v_2)] = zf(z,t)$ satisfies the property of the sector $zf(z,t) \geq 0 \ \forall z \in R$.

Taking into account the above result, the error dynamics in (11) can be rewritten as:

$$\dot{e}(t) = (A - LC)e(t) + G.f(z,t) \quad (12)$$

$$z = (H - KC)e(t) \quad (13)$$

Relations (12)-(13) show, once again, that the error dynamics can then be considered as a linear system controlled by a time-varying nonlinearity function $f(z,t)$ satisfying the sector property. Circle criterion establishes that the feedback interconnection of a linear system and a time-varying nonlinearity satisfying the sector property is globally uniformly asymptotically stable [5], [6]. Advantages of the circle-criterion approach are the global Lipschitz restrictions removing and high gain avoiding. However it introduces linear matrix inequality (LMI) conditions. An extension to multi-variable discrete-time case is given in [7] for systems with multiple nonlinearities.

Based upon the error dynamics, relation (12)-(13), the nonlinear observer design problem is then equivalent to stabilization of the error dynamics problem. To this end a candidate Lyapunov function $V = e^T Pe$ is considered. In order to ensure asymptotic stability of the observer the derivative of the candidate Lyapunov function must be negative or null.

$$\dot{V} = \dot{e}^T Pe + e^T P\dot{e} \quad (14)$$

With the help of relation (12) and (13) the derivative of the Lyapunov function becomes:

$$\begin{aligned}\dot{V} = e^T &\left[(A - LC)^T P + P(A - LC)\right]e + \\ &+ f^T(z,t)G^T Pe + e^T PGf(z,t)\end{aligned} \quad (15)$$

By setting:

$$(A - LC)^T P + P(A - LC) \leq -Q \quad (16)$$

and $PG = -(H - KC)^T$ (17)

With $Q = \varepsilon I_n$ and $\varepsilon \succ 0$, the derivative of the Lyapunov function can be rewritten as:

$$\dot{V} \leq -e^T Qe - 2.z^T.f(z,t) \quad (18)$$

Thus ends the proof.

Note that the existence of observer (9)-(10) is conditioned by the solution of LMI conditions (7)-(8). By solving LMI constraints, observer gain matrices L and K that guarantee observer convergence are then computed. Restriction of the sector property ensures that the vector time-varying nonlinearity in the observer error system satisfies the sector condition of the circle criterion [5], [6]. In Ibrir [7], the author has investigated the study of globally Lipschitz systems and bounded-state nonlinear systems. Bounded-state nonlinear systems constitute a large class of system that includes electric machine systems. Electric machine models involve the magnetic flux as a key and bounded state variable that combined with other state variable of the machine, such as rotor angular velocity, leads to the nonlinear part of the machine model. This is due to the effect of the magnetic material saturation property that is similar to the sector nonlinearity.

3. INDUCTION MOTOR NONLINEAR MODEL

Induction motor as various electric machines constitutes a theoretically interesting and practically important class of nonlinear dynamic systems. Induction motor is known as a complex nonlinear system in which time-varying parameters entail additional difficulty for induction motor system control and conditions monitoring. Based on the fact that the nonlinear model of the induction motor system can be significantly simplified, if one applies the d-q Park transformation, different structures of the nonlinear model are investigated and discussed in [1]. In this paper the induction motor nonlinear model is described, in stator fixed d-q Park reference frame, by the following nonlinear differential equations with, the stator current, rotor flux and rotor angular velocity as selected state variables of the machine.

$$\frac{d}{dt} i_{sd} = -\gamma i_{sd} + \frac{\beta}{T_r} \varphi_{rd} + \beta \omega_r \varphi_{rq} + \frac{1}{\sigma l_s} u_{sd} \quad (19)$$

$$\frac{d}{dt} i_{sq} = -\gamma i_{sq} - \beta \omega_r \varphi_{rd} + \frac{\beta}{T_r} \varphi_{rq} + \frac{1}{\sigma l_s} u_{sq} \quad (20)$$

$$\frac{d}{dt} \varphi_{rd} = \frac{m}{T_r} i_{sd} - \frac{1}{T_r} \varphi_{rd} - \omega_r \varphi_{rq} \quad (21)$$

$$\frac{d}{dt} \varphi_{rq} = \frac{m}{T_r} i_{sq} + \omega_r \varphi_{rd} - \frac{1}{T_r} \varphi_{rq} \quad (22)$$

$$\frac{d}{dt} \omega_r = \alpha(\varphi_{rd} i_{sq} - \varphi_{rq} i_{sd}) - k_f \omega_r - k_l T_l \quad (23)$$

Where $\alpha = \frac{n_p^2 m}{J l_r}$, $\beta = \frac{1}{m}\left(\frac{1-\sigma}{\sigma}\right) = \frac{1}{\sigma}\frac{m}{l_s l_r}$, $\gamma = \frac{1}{\sigma}\left(\frac{1}{T_s} + \frac{1-\sigma}{T_r}\right)$, $\sigma = 1 - \frac{m^2}{l_s l_r}$, $k_f = \frac{f_r}{J}$, $k_l = \frac{n_p}{J}$, and $\omega_r = n_p \Omega_r$.

The indexes s and r refer to the stator and the rotor components respectively and the indexes d and q refer to the direct and quadrature of the fixed stator reference frame components respectively (Park's vector components). i and u are the current and voltage vector, φ is the flux vector, r is the resistance, l is the inductance, m is the mutual inductance. T_s and T_r are the stator and the rotor time constant respectively. ω_r is the rotor angular velocity, f_r is the friction coefficient, J is the moment of inertia coefficient, n_p is the number of pair poles, Ω_r is the mechanical speed of the rotor and finally T_l is the mechanical load torque.

The considered induction motor system model has three inputs, and only two state variables available for measurement which are the stator current components. The nonlinearity of the model is mainly introduced by the product of the rotor angular velocity and the rotor flux components in the four first relations and the torque in the fifth relation as the product of two state variables namely the stator current components and the rotor flux components. In order to take into account certain of the time-varying parameters, as stator (rotor) resistance, one has to introduce an additional equation relating to the considered parameter variation. Thus leads to a state space model of six state variable dimensions. In this paper we consider only the nonlinearity effect introduced by the variation of the rotor angular velocity.

This type of nonlinear model is generally used for performing nonlinear control, conditions monitoring and faults diagnosis of electric induction machine systems. Performing these techniques requires estimating unmeasured rotor flux linkage and rotor angular velocity state variables based on the stator current and voltage measurements. In this context, the circle criterion approach application is investigated to design a nonlinear observer for the machine sensorless control purpose. To satisfy sector conditions (1)-(4) nonlinearities of the machine model (19)-(23) are function of the flux state variable that is a bounded state variable. The nonlinearities of the model are of the form $\omega_r \varphi_{rd}$ that can be expressed as:

$$\omega_r \varphi_{rd} = (\omega_r \varphi_{rd} + \rho \omega_r) - \rho \omega_r \quad (24)$$

One can verify that:

$$\frac{\partial}{\partial \omega_r}(\omega_r \varphi_{rd} + \rho \omega_r) = \varphi_{rd} + \rho \geq 0 \quad (25)$$

With $\|\varphi_{rd}\| \leq 2$, then one can choose $\rho = 2$.

Once again the system nonlinearity is decomposed into a nonlinearity satisfying the sector property and a linear part to be added to the linear part of the system expressed in the evolution matrix A.

4. Simulation Results and Comments

Characteristics of the considered induction machine are listed in Table 1.

The first step of the simulation consists of resolving the LMI conditions, relation (7)-(8), using an adequate LMI tools such as the LMI tool-box of the Matlab software. The obtained nonlinear observer gain matrices L and K are the following:

$$L = \begin{bmatrix} -132.3581 & -0.0000 \\ 0.0000 & -132.3581 \\ 1.7914 & -0.0000 \\ 0.0000 & 1.7914 \\ -0.0000 & 0.0000 \end{bmatrix}$$

$$K_1 = [5.4133 \quad -3.0149],\ K_2 = [3.0149 \quad -5.4133],$$
$$K_3 = [-4.0085 \quad 5.0085],\ K_4 = [5.0085 \quad -4.0085]$$

The corresponding Lyapounov matrix for this LMI feasibility test is:

$$P=\begin{bmatrix} 0.1787 & -0.0995 & 0.0029 & -0.0003 & -0.0330 \\ -0.0995 & 0.1787 & -0.0003 & 0.0029 & 0.0330 \\ 0.0029 & -0.0003 & 0.0871 & -0.0080 & -0.0000 \\ -0.0003 & 0.0029 & -0.0080 & 0.0871 & 0.0000 \\ -0.0330 & 0.0330 & 0.0000 & -0.0000 & -0.0000 \end{bmatrix}$$

And $\varepsilon = 0.04$.

TABLE I

Characteristics of the Considered Induction Machine

Symbol	Quantity	Numerical value SI [a]
P	Power	1.5 KW
f	Supply frequency	50 Hz
n_p	Number of pair poles	2
u	Supply Voltage	220 V
R_s	Stator resistance	4.85 Ω
R_r	Rotor resistance	3.805 Ω
m	Mutual Inductance	0.258 H
l_s	Stator Inductance	0.274H
l_r	Rotor Inductance	0.274 H
ω_r	Rotor angular speed	297.25 rd/s
k_f	Friction Coefficient	0.00114 N.s/rd
J	Inertia Coefficient	0.031 Kg^2/s
T_l	Load torque	5 Nm

The second step of simulation consists of injecting the obtained numerical values of the gain matrix L and the vectors K_i in an S-function-based Matlab program that interacts with the Matlab Simulink software to simulate the nonlinear system and the nonlinear observer as shown in Fig.1.

The simulation results of the designed nonlinear observer are presented in the following. Fig.2 and Fig.3 show the measured and estimated stator current and rotor flux components respectively. Fig.4 and Fig.5 show the measured and estimated rotor angular velocity and the corresponding load torque respectively with corresponding estimation error. One can see that the estimated state variables of the machine follow the desired trajectories.

To highlight these results a load torque is introduced in the simulation at time of 1.5 sec., the simulation results show that all the state variables of the machine are modified accordingly. Thus demonstrate that the effectiveness of the circle criterion based nonlinear observer design for the induction machine system state estimation.

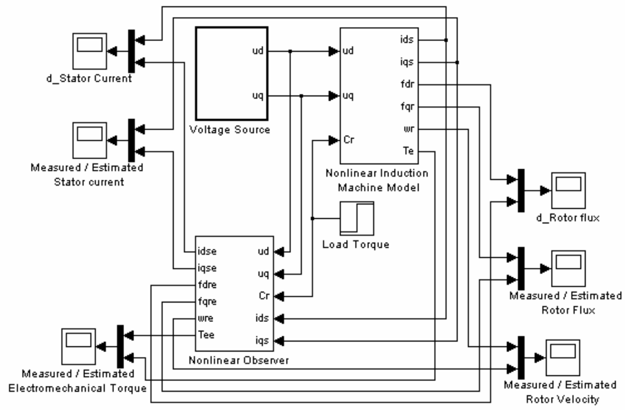

Fig.1: Matlab-Simulink Simulation Scheme

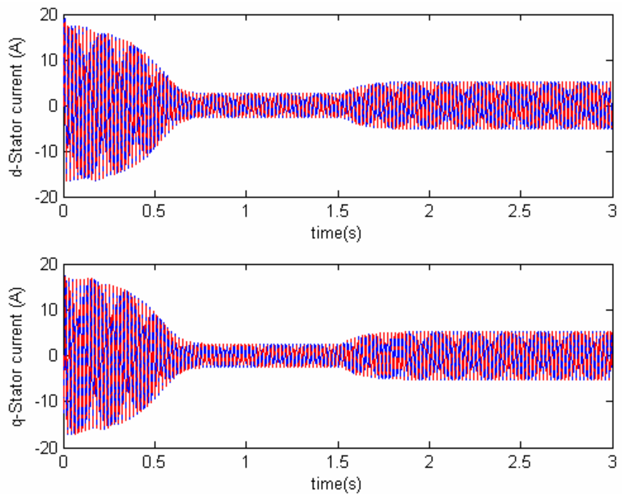

Fig.2: Measured and observed stator current components

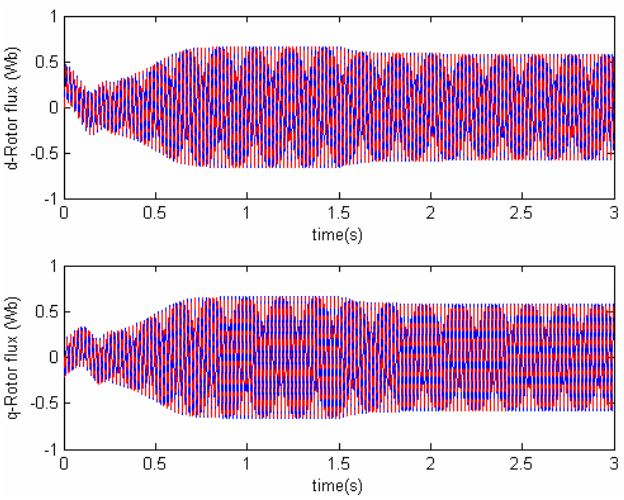

Fig.3: Measured and observed rotor flux components

Measured and estimated state variables of the considered induction machine can be used to control the machine system via an adequate state feedback control of the machine. Evaluation of the performance of the designed observer in the low speed region and even at zero speed, with and without, load torque that effectively opens a large variety of applications of sensorless induction motor drive, will be investigated in the forthcoming paper.

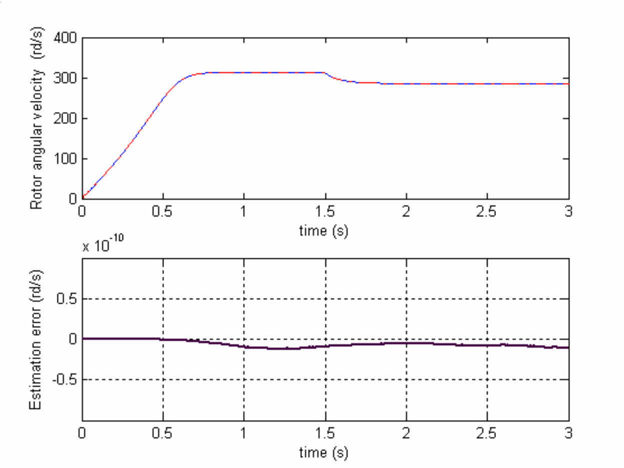

Fig.4: Measured and observed rotor angular velocity

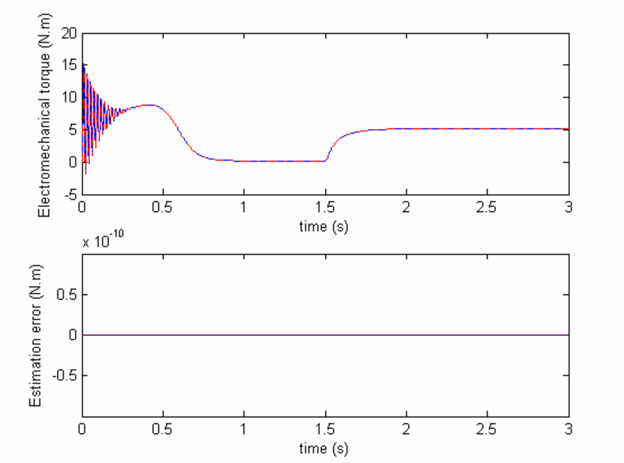

Fig. 5: Measured and estimated electromechanical torque of the machine

5. Conclusion

A circle criterion based nonlinear observer design for induction motor sensorless control has been presented. The main advantage of the circle criterion approach is that it permits to exploit directly the nonlinearities of the system without attempt to eliminate them. Linear matrix inequalities are solved to determine the gain matrices of the nonlinear observer. A nonlinear induction machine model with stator current, rotor flux, and rotor angular velocity as selected state variable has been simulated. Simulation results show that the circle criterion based nonlinear observer design can effectively be performed for induction motor sensorless control.

References

[1] B. Bensaker, H. Kherfane, A. Maouche and R. Wamkeue, Nonlinear modeling of induction motor drives for nonlinear sensorless control purposes. *Preprints 6th IFAC Symposium on Nonlinear Control Systems*. Stuttgart, Germany. Vol.3. Sep. 01-03, 2004, 1475-1480.

[2] A. J. Krener and A. Isidori, Linearization by output injection and nonlinear observers. *Systems & Control Letters,* (3), 1983, 47-52.

[3] A. Isidori, Nonlinear control systems: An introduction. *Lecture Notes in Control and Information Sciences. Springer-Verlag*, 1985, 178-253.

[4] A. N. Atassi and H. K. Khalil, Separation results for the stabilization of nonlinear systems using different high-gain observer designs. *Systems & Control Letters*. 39, 2000, 183-191.

[5] M. Arcak and P. Kokotovic, Nonlinear observers: a circle criterion design and robustness analysis. *Automatica*. Vol. 37, (12), 2001, 1923-1930

[6] M. Arcak, Certainty-equivalence output-feedback design with circle-criterion observers. *IEEE Transactions on Automatic Control,* Vol.50, (6), June 2005, 905-909.

[7] S. Ibrir, Circle-criterion approach to discrete-time nonlinear observer design. *Automatica,* 43, 2007,1432 – 1441

[8] M. Bodson and J. Chiasson: Differential-geometric methods for control of electric motors. *International Journal of Robust and Nonlinear Control.* Vol.8, 1998, pp.923-954.

[9] B. Castillo-Toledo, S. Di Gennaro, A. G. Loukianov and J. Rivera: Discrete-time sliding mode control with application to induction motors. *Automatica*. Vol.44, 2008, pp.3036-3045.

[10] M. G. Campbell, J. Chiasson, M. Bodson and L. M. Tolbert: Speed sensorless identification of the rotor time constant in induction machines. *IEEE Transactions on Automatic Control.* Vol. 52, (4), 2007, pp.758-763.