Nonlinear Control of Induction Motor: A Combination of Nonlinear Observer Design and Input-Output Linearization Technique

Farid Berrezzek, Wafa Bourbia and Bachir Bensaker

Abstract--**This paper deals with a nonlinear control strategy of induction motor that combines an input-output linearization control technique and a nonlinear observer design. It is well known that induction motors are the most widely used motors in electrical appliances, industrial control, and automation. However, it is also known that induction motor control is a complex task that is due to its nonlinear characteristics. Two main features of the proposed approach are worth to be mentioned. Firstly a nonlinear control is carried out using a nonlinear feedback linearization technique involving non available state variable measurements of the induction motor system. Secondly a nonlinear observer is designed to estimate these pertinent but unmeasurable state variables of the machine. The circle-criterion approach is performed to compute the observer gain matrices as a solution of linear matrix inequalities (LMI) that ensure the stability conditions of the estimated state error dynamics of the designed observer in the sense of Lyapunov. Simulation results are presented to validate the effectiveness of the proposed approach**

Index Terms---- **Induction motor; Nonlinear control; Input-output linearization; Nonlinear observer; Circle criterion; LMI ; Lyapunov stability.**

I. Introduction

INDUCTION motors are suitable electromechanical systems for a large spectrum of industrial applications. However, induction motors are multivariable nonlinear and strongly coupled time-varying systems, mainly, in variable speed applications. Thus makes their control so difficult [1]. In the majority of industrial applications, it is necessary to be able to control the speed of induction motor drives. The most common technique to achieve this task is the well known vector control technique that requires a speed sensor which is usually placed on the rotor shaft of the machine. The speed sensor has some disadvantages such as being costly and it reduces the robustness and reliability of the induction motor. Due to significant influence of nonlinearities on induction motor system dynamics, the linear control techniques are quite good and they may not meet the system specifications mainly in the case of variable speed applications [2]. Consequently, this has opened a new and interesting area for academic research and industrial applications for nonlinear control techniques. During the last few years, a variety of solutions that have promoted the market of control techniques became industrial standard for medium and low performance applications. Among these nonlinear control techniques that ensure high performance and global decoupling between the outputs to control whatever the path profile imposed for the machine, one can mention the input-output feedback linearization technique developed by Isidori [3]. This method implements the differential geometry theory to transform a nonlinear system into a linear one by using a state feedback linearizing method that ensure input-output decoupling, and after that it applies a method of linear system control theory. The availability of powerful low-cost microprocessors has spurred great advances in the theory and application of nonlinear control techniques as sliding mode, nonlinear predictive and nonlinear adaptive control techniques [4], [5], [6], [11]. All these methods assume that all state variable measurements of the considered system are available online.

In order to implement a nonlinear sensorless control technique, it is necessary to synthesize an observer for the estimation of non-measurable state variables of the machine system that are essential for control purposes. In this paper a state feedback linearizing controller is used in combination with a nonlinear state observer designed via the circle criterion approach for induction motor control. Circle criterion approach for nonlinear observer design has been introduced for the first time by Arcak and Kokotovic for systems that can be decomposed into a linear part and a non-linear part [7], [8]. The observer gain matrices are determined as a solution of linear matrix inequalities (LMI) that ensure the global asymptotic convergence of the observer dynamics. The main advantage of circle criterion approach is that it exploits directly the system nonlinearity properties with minimal restrictions in contrast of the other methods which attempt to eliminate or at least to diminish their effects.

The paper is organized as follows: in the second section we present the considered induction motor nonlinear model. In the third section we recall the input-output linearization control technique and in the forth section the circle-criterion based nonlinear observer design is presented. The simulation results and comments are presented in the fifth and final section.

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II. NONLINEAR INDUCTION MOTOR MODEL

Induction motor as various electric machines constitutes a theoretically interesting and practically important class of nonlinear dynamic systems. Induction motor is known as a complex nonlinear system in which time-varying parameters entail additional difficulty for induction motor system control, conditions monitoring and faults diagnosis. Based on the fact that the nonlinear model of the induction motor system can be significantly simplified, if one applies the d-q Park transformation, different structures of the nonlinear model are investigated and discussed in [10]. The choice of a model depends on measurement possibilities, selected state variables of the machine and the problem at hand. In this paper, the considered induction motor model has stator current, rotor flux and rotor angular velocity as selected state variables. The control inputs are the stator voltage and load torque. The available stator current measurements are the induction motor system outputs. The nonlinear state space model of the induction motor is expressed as the following:

$$\frac{d}{dt}i_{sd} = -\gamma i_{sd} + \frac{\beta}{T_r}\varphi_{rd} + \beta\omega_r\varphi_{rq} + \frac{1}{\sigma l_s}u_{sd} \quad (1)$$

$$\frac{d}{dt}i_{sq} = -\gamma i_{sq} - \beta\omega_r\varphi_{rd} + \frac{\beta}{T_r}\varphi_{rq} + \frac{1}{\sigma l_s}u_{sq} \quad (2)$$

$$\frac{d}{dt}\varphi_{rd} = \frac{m}{T_r}i_{sd} - \frac{1}{T_r}\varphi_{rd} - \omega_r\varphi_{rq} \quad (3)$$

$$\frac{d}{dt}\varphi_{rq} = \frac{m}{T_r}i_{sq} + \omega_r\varphi_{rd} - \frac{1}{T_r}\varphi_{rq} \quad (4)$$

$$\frac{d}{dt}\omega_r = -k_f\omega_r + \alpha T_e - k_l T_l \quad (5)$$

$$T_e = n_p\frac{m}{L_r}\left(\varphi_{rd}i_{sq} - \varphi_{rq}i_{sd}\right) \quad (6)$$

With : $\alpha = \frac{n_p^2 m}{J l_r}, \beta = \frac{1}{m}\left(\frac{1-\sigma}{\sigma}\right) = \frac{1}{\sigma}\frac{m}{l_s l_r}$,

$\sigma = 1 - \frac{m^2}{l_s l_r}$, $k_f = \frac{f_r}{J}$, $k_l = \frac{n_p}{J}$, and $\omega_r = n_p\Omega_r$.

The indexes s and r refer to the stator and the rotor components respectively and the indexes d and q refer to the direct and quadrature of stator fixed reference frame components respectively (Park's vector components). i and u are the current and voltage vector, φ is the flux vector, r is the resistance, l is the inductance, m is the mutual inductance. T_s and T_r are the stator and the rotor time constant respectively. ω_r is the rotor angular velocity, f_r is the friction coefficient, J is the moment of inertia coefficient, n_p is the number of pair poles, Ω_r is the mechanical speed of the rotor and finally T_l is the mechanical load torque.

One can see that nonlinearities of the model are expressed by the product between the state variables of the machine system as stator current and rotor flux components in the electromechanical torque expression and rotor flux and rotor angular velocity in the other equations. In this type of nonlinear model only the stator voltage and current measurements are available. Rotor flux and angular velocity are essential for induction motor control and conditions monitoring need to be estimated with the help of a state observer.

The above nonlinear state space model can be rewritten in the following form:

$$\dot{x}(t) = Ax(t) + \phi(y(t), u(t)) + Gf[H.x(t)] \quad (7)$$

$$y(t) = Cx(t) \quad (8)$$

Where $x(t)$ represents the state vector of the machine, and $u(t)$ and $y(t)$ are the system inputs and outputs respectively. A, G, C and H are known constant matrices with appropriate dimensions. $\phi(u, y)$ is an arbitrary real-valued vector that depends only on the system inputs and outputs and finally the $f[.]$ represents the system nonlinearities. One can see that the above nonlinear model is composed of a linear part and a nonlinear part.

III. INPUT-OUTPUT LINEARIZING CONTROLLER DESIGN FOR INDUCTION MOTOR

The concept of input-output feedback linearization is to find a coordinate system transformation such that the nonlinear dynamics represented in the new coordinate system can be canceled by state feedback [3], [5], [6]. This is a quite appealing approach when such a transformation exists. The system to be controlled, through a linearization control law, must be a square type [4], [5]. In the case of induction motor application one can, therefore, choose the rotor speed and the norm of the rotor flux as system outputs. The choice of the squared norm is justified only by calculus simplification purpose. Taking into account the above conditions, the outputs of our induction motor system are then expressed in a vector form as follows:

$$y_1 = h_1(x) = \omega_r$$
$$y_2 = h_2(x) = \varphi_{rd}^2 + \varphi_{rq}^2 = \varphi_r^2 \quad (9)$$

or $y = h(x) = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ (10)

Where $h(x)$ is an analytic function and the state vector $x(t)$ belongs to the following set.

$$\Omega = \left\{x \in \Re^5 : \varphi_{rd}^2 + \varphi_{rq}^2 \neq 0\right\} \quad (11)$$

The Lie directional derivative of the analytic function $h(x)$ with respect to the field vector $f(x)$ is defined as follows [5], [6]:

$$L_f h_j(x) = \sum_{i=1}^{n} \frac{\partial h_j}{\partial x_i} f_i(x) \qquad (12)$$

Repeatedly, we have:

$$L_f^i h_j = L_f (L_f^{i-1} h_j) \qquad (13)$$

The time derivative of the system output y_j can be expressed as :

$$\dot{y}_j = L_f h_j + \sum_{i=1}^{p} (L_{gi} h_j) u_i \qquad (14)$$

Where p is the number of outputs.

The relative degree of the nonlinear system (1)-(5), affine in control, is the vector $(r_1, r_2, ..., r_p)$ verifying the existence of at least one derivative such as:

$$L_{gi} L_f^{rj-1} h_j(x) \neq 0 \qquad (15)$$

The element r, is the first derivative of y_j showing explicitly the control u in its expression as:

$$y_j^{rj} = L_f^{rj} h_j + \sum_{i=1}^{p} (L_{gi} L_f^{rj-1} h_j) u_i \qquad (16)$$

Note that for a controllable system, we always have $r \leq n$; with « n » as the system order.
The total relative degree is defined as the sum of all the relative degrees, it must be less than or equal to the system order [3]. In these conditions we say that the above system has a relative degree at the state x_0.
Using this type of procedure in the case of the induction motor system, it is easy to verify that the control appears for the first time in the second derivative of the outputs y1 and y2 as one can see in the following:

$$\dot{y}_1 = L_f h_1$$
$$\dot{y}_2 = L_f h_2$$
$$\ddot{y}_1 = L_f^2 h_1 + L_{g11} L_f h_1 u_{sd} + L_{g12} L_f h_1 u_{sq}$$
$$\ddot{y}_2 = L_f^2 h_2 + L_{g11} L_f h_2 u_{sd} + L_{g12} L_f h_2 u_{sq}$$

The derivative of the second order can be rewritten in the matrix form as:

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} L_f^2 h_1 \\ L_f^2 h_2 \end{bmatrix} + \begin{bmatrix} L_{g11} L_f h_1 & L_{g12} L_f h_1 \\ L_{g11} L_f h_2 & L_{g12} L_f h_2 \end{bmatrix} \begin{pmatrix} u_{sd} \\ u_{sq} \end{pmatrix} \qquad (17)$$

This matrix form can be written as:

$$\begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{pmatrix} = \begin{pmatrix} V_d \\ V_q \end{pmatrix} = B(x) + A(x) \begin{pmatrix} u_{sd} \\ u_{sq} \end{pmatrix} \qquad (18)$$

The relative degree of the induction motor system is (2, 2), and verify $r \leq n$. From relation (18) the control law can be expressed as:

$$\begin{pmatrix} u_{sd} \\ u_{sq} \end{pmatrix} = A^{-1}(x) \left[\begin{pmatrix} V_d \\ V_q \end{pmatrix} - B(x) \right] \qquad (19)$$

Where the vector $V=[Vd \;\; Vq]^T$ is an external set of linearized system and the term $A(x)$ is the matrix of input-output decoupling such as:

$$\det[A(x)] = -2p \frac{m}{\sigma^2 l_s^2 l_r} \cdot \frac{1}{T_r} (\varphi_{rd}^2 + \varphi_{rq}^2) \qquad (20)$$

From the above equation one can see that the state feedback control decouples and linearizes the induction motor system model. Therefore, the closed loop system is equivalent to two independent chains of two integrators, relation (18). The control vector V can be computed as follows:

$$\begin{cases} V_d = -k_{a1}(\omega_r - \omega_{ref}) - k_{a2}(\dot{\omega}_r - \dot{\omega}_{ref}) + \ddot{\omega}_{ref} \\ V_q = -k_{b1}(\varphi_r^2 - \varphi_{ref}^2) - k_{b2}(\dot{\varphi}_r^2 - \dot{\varphi}_{ref}^2) + \ddot{\varphi}_{ref}^2 \end{cases} \qquad (21)$$

Note that the involved state variables of the machine system are unmeasurable and therefore they must be estimated with the help of an observer design technique. The retained technique in this paper is the circle-criterion approach. To this end $y_r = [\omega_{ref} \;\; \varphi_{ref}^2]^T$ is selected as a reference output trajectory and $y = (\omega_r \;\; \varphi_r^2)^T$ as the estimated output. This choice leads to the following tracking error dynamics:

$$\ddot{e}_1 + k_{a2}\dot{e}_1 + k_{a1} e_1 = 0$$
$$\ddot{e}_2 + k_{b2}\dot{e}_2 + k_{b1} e_2 = 0 \qquad (22)$$

Where $e_1 = \omega_r - \omega_{ref}$ and $e_2 = \varphi_r^2 - \varphi_{ref}^2$ and k_{ai} , k_{bi} are respectively the coefficients of Hurwitz polynomials.

IV. NONLINEAR OBSERVER DESIGN

In contrast of linearization techniques and high gain approaches, which attempt to eliminate and to dominate the system nonlinearities in designing a nonlinear observer, circle-criterion approach exploits the type of system nonlinearities. In its basic form, circle criterion approach for nonlinear observer design, introduced by Arcak and Kokotovic [7], is applicable to a class of systems that can be decomposed in linear and nonlinear parts with a condition that the nonlinear part is a time-varying function that satisfies the following sector property.

A. Definition (Sector property)

A memoryless function $f(z,t):[0\ +\infty[\times R^p \rightarrow R^p$ is said to belong to the sector $[0\ +\infty[$ if $zf(z,t) \geq 0$.

Let v_1 and v_2 be two real positive numbers. By setting $z = v_1 - v_2$ and $f(z,t) = [f(v_1,t) - f(v_2,t)]$, the above sector property is equivalent to:

$$(v_1 - v_2)[f(v_1,t) - f(v_2,t)] \geq 0 \ \forall v_1, v_2 \in R^+ \qquad (23)$$

Relation (23) states that the nonlinear function $f(z,t)$ is a nondecreasing function. Furthermore, if $f(z,t)$ is a continuously differentiable function the above relation is also equivalent to [7], [8]:

$$\frac{d}{dz} f(z,t) \geq 0 \quad \forall z \in R \qquad (24)$$

In the following we recall the main theorem and conditions that are used in this work to study the feasibility of nonlinear observer design for induction motor control with respect to circle criterion or sector property.

B. Theorem ([7], [8], [9])

Consider a nonlinear system of the form (7)-(8) with the nonlinear part satisfying the circle criterion relations (23)-(24). If there exist a symmetric positive definite matrix $P \in R^{nxn}$, a set of row vectors $K \in R^p$ and a small positive real constant $\varepsilon \succ 0$ such that the following linear matrix inequalities (LMI) hold:

$$(A - LC)^T P + P(A - LC) + \varepsilon I_n \leq 0 \qquad (25)$$

$$PG + (H - KC)^T = 0 \qquad (26)$$

Then a nonlinear observer can be designed as:

$$\begin{aligned} \dot{\hat{x}}(t) = A\hat{x}(t) + \phi[u(t), y(t)] + L[y(t) - \hat{y}(t)] + \\ + G f[H\hat{x}(t) + K(y(t) - \hat{y}(t))] \end{aligned} \qquad (27)$$

$$\hat{y}(t) = C\hat{x}(t) \qquad (28)$$

And $\lim_{t\to\infty} e(t) = x(t) - \hat{x}(t) = 0$.

Where $\hat{x}(t)$ and $\hat{y}(t)$ are the estimate of the state $x(t)$ and the output $y(t)$ of the nonlinear system respectively. L and K are the observer gain matrices to be determined such as the state error dynamics are asymptotically stable in the sense of Lyapunov.

One can see that the structure of the designed nonlinear observer is similar to Luenberger linear observer with an additional term that represents the system time-varying nonlinearities. State estimation error dynamics, which are the difference between the dynamics of the system motor model and the dynamics of the observer, can be expressed as:

$$\dot{e}(t) = (A - LC)e(t) + G.f(z,t) \qquad (29)$$

$$z = (H - KC)e(t) \qquad (30)$$

Where $e(t) = x(t) - \hat{x}(t)$ is the state estimation error.

One can see, once again, that state estimation error dynamics, relations (29)-(30), are composed of a linear part and a time-varying nonlinearity that satisfies the sector property. Circle criterion establishes that this type of interconnection system is globally uniformly asymptotically stable [7], [8].

To proof the stability of the observer error dynamics a Lyapunov candidate function, $V = e^T Pe$, is used. Negativity conditions for the time derivative of the Lyapunov function determine the linear matrix inequalities (LMI) conditions, relations (25)-(26). Resolution of these LMI conditions leads to feasible values of the observer gain matrices *L* and *K*. An extension to multivariable systems with multiple nonlinearities is presented in [9].

V. Simulation Results and Comments

In order to illustrate the performance of the proposed controller we provide a series of simulations. The Characteristics of the considered induction machine are listed in Table 1.

TABLE 1
CHARACTERISTICS OF THE CONSIDERED INDUCTION MACHINE

Symbol	*Quantity*	*Numerical Value*
P	Power	1..5 Kw
f	Supply frequency	50 Hz
np	Number of pair poles	2
u	Supply Voltage	220 V
Rs	Stator resistance	4.85 Ω
Rr	Rotor resistance	3.805 Ω
Ls	Stator Inductance	0.274 H
Lr	Rotor Inductance	0.274 H
m	Mutual Inductance	0.258 H
ω_r	Rotor angular speed	297.25 rd/s
k_f	Friction Coefficient	0.00114 N.s/rd
J	Inertia Coefficient	0.031 Kg^2/s
Tl	Load torque	5 N.m
ka1,ka2, kb1,kb2	Coefficients of Hurwitz polynomials	10^5 - 1000 10^5 - 1000

The first step of the simulation consists of resolving the LMI conditions, relation (25)-(26), using an adequate LMI tools such as the LMI toolbox of the Matlab software. The obtained nonlinear observer gain matrices L and K are the following:

$$L=\begin{bmatrix}-132.3581 & -0.0000\\ 0.0000 & -132.3581\\ 1.7914 & -0.0000\\ 0.0000 & 1.7914\\ -0.0000 & 0.0000\end{bmatrix}$$

$$K_1=[5.4133\ \ -3.0149],\ K_2=[3.0149\ \ -5.4133],$$
$$K_3=[-4.0085\ \ 5.0085],\ K_4=[5.0085\ \ -4.0085]$$

The corresponding Lyapounov matrix for the LMI feasibility test is:

$$P=\begin{bmatrix}0.1787 & -0.0995 & 0.0029 & -0.0003 & -0.0330\\ -0.0995 & 0.1787 & -0.0003 & 0.0029 & 0.0330\\ 0.0029 & -0.0003 & 0.0871 & -0.0080 & -0.0000\\ -0.0003 & 0.0029 & -0.0080 & 0.0871 & 0.0000\\ -0.0330 & 0.0330 & 0.0000 & -0.0000 & -0.0000\end{bmatrix}$$

With $\varepsilon = 0.04$.

The simulation block diagram of circle criterion nonlinear observer to estimate the needed state variables of the machine system is given in figure 1.

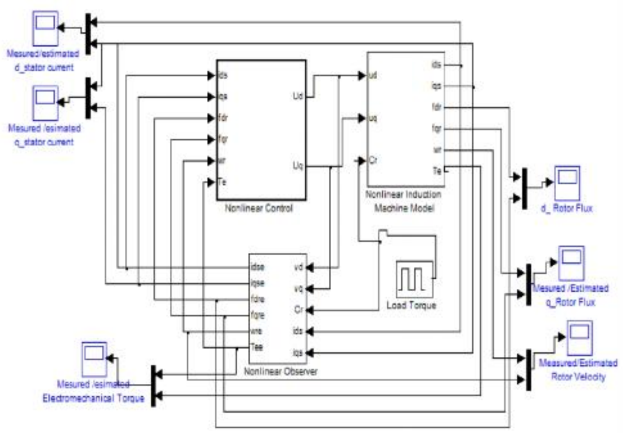


Fig. 1. Simulation block scheme

Once the estimated state variables of the machine system are available the input-output state feedback control is implemented.The simulation results are presented in the following figures.

From figure 2 to figure 6, we present the estimated and measured state variables of the machine according to load torque variation from no load value to the value Cr= 5 N.m introduced at time t=0.5 s and return to no load value, and rotor angular speed variations form the reference value w_ref = 120 [rd/s] to w_ ref = -120 [rd/s] at time t= 4 s.

In figure 3 and figure 5 one can see a significant decoupling effect of flux components under rotor angular speed and load torque variations in d-q Park representation.

Analysis of the simulation results shows that the obtained performance of rotor angular speed and flux tracking are very adequate. Analysis of the different figures points out that designed nonlinear observer effectively estimates the unmeasured state variables of the machine and tracks the load torque variations with respect to applied nonlinear control law computed in accordance with the input-output linearization feedback control technique.

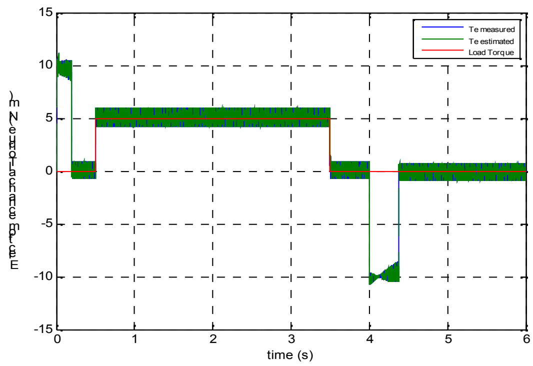


Fig. 2. Measured and estimated electromechanical torque

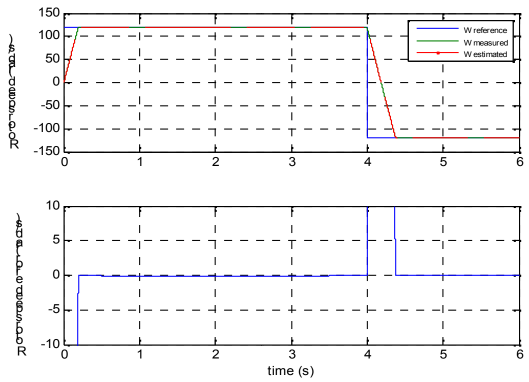
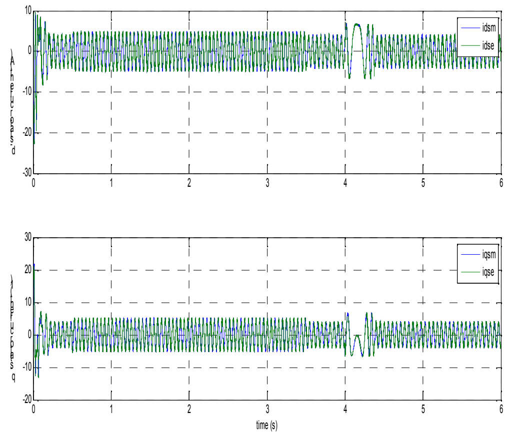


Fig. 3. Rotor speed evolution according to load variations

VI. Conclusions

In this paper we have presented a nonlinear control of induction motor by using a nonlinear observer designed based on circle-criterion approach and input-output linearization technique. Simulation results show that this control strategy assures a perfect linearization regardless profile trajectories physically imposed on the asynchronous machine. Decoupling is achieved between the two selected outputs (speed and flux) with the help of the proposed approach. The designed nonlinear observer has contributed effectively to estimate the unmeasurable state variables that are essential for the nonlinear control. Simulation results show that this approach improves the performance of trajectory tracking and should bypass shortcomings of conventional methods. To this end experimental tests will be investigated in a future framework.


idsm
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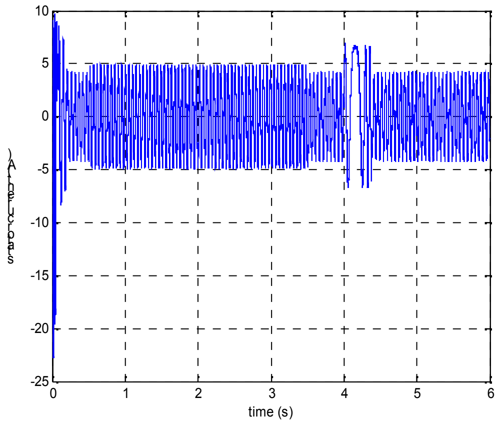
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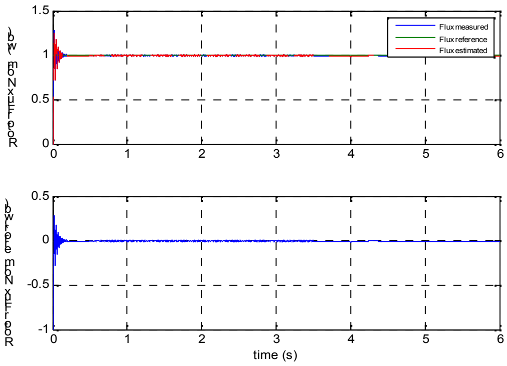
Flux measured
Flux reference
Flux estimated
time (s)