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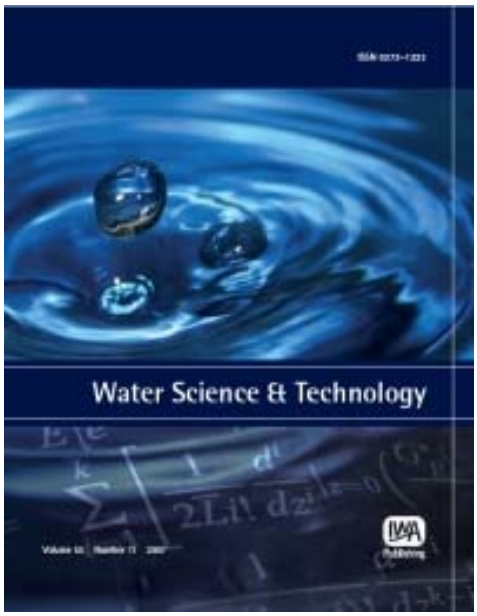

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New equation for the computation of flow velocity in partially filled pipes arranged in parallel

Lotfi Zeghadnia, Lakhdar Djemili, Larbi Houichi and Nouredin Rezgui

ABSTRACT

This paper presents a new approach for the computation of flow velocity in pipes arranged in parallel based on an analytic development. The estimation of the flow parameters using existing methods requires trial and error procedures. The assessment of flow velocity is of great importance in flow measurement methods and in the design of drainage networks, among others. In drainage network design, the flow is mostly of free surface type. A new method is developed to eliminate the need for trial methods, where the computation of the flow velocity becomes easy, simple, and direct with zero deviation compared to Manning equation results and other approaches such as that have been considered as the best existing solutions. This research work shows that these approaches lack accuracy and do not cover the entire range of flow surface angles: $0^\circ \leq \theta \leq 360^\circ$.

Key words | circular pipe, flow velocity, free surface flow, Manning equation, uniform and steady flow

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INTRODUCTION

Flow velocity calculation is an important task for hydraulic engineers, especially in the design of open channel conveyance for irrigation, drainage, and sewer networks. Flow in the latter is usually under free surface conditions, in order to avoid a decrease of the water cross-sectional area and thereby the decrease of the flow efficiency. The Manning model is considered as the best model to describe free surface flow. Several authors, such as Chow (1959), Henderson (1966), Metcalf & Eddy Inc. (1981), Carlier (1980), Swamee (1994), Swamee & Rathie (2004), and Hager (2010), have discussed the model at length.

We considered in this study pipes with a circular cross-section as these are the preferred form for sewer system design. Subwatersheds can be arranged in series or in parallel. Similarly, pipes in a sewer or drainage system can be arranged in series or in parallel. Using Manning's equation, the computation of the flow velocity and water surface angle is not direct and needs to go through trial methods with heavy computations. Based on the Manning model, several researchers have been attempting to propose an explicit solution for free surface flow computation. Among these are Saatçi (1990), Giroud *et al.* (2000), and Akgiray (2004, 2005) who have tried to eliminate the need for iterative and trial methods for water surface angle ranging between 0° and 302.41°.

Transition into a pressurised flow regime may occur during intense rain events as inflow exceeds the transport capacity of the system in free flow mode. A number of authors considered that it is reliable to simulate flow in a pressurised pipe as surface flow using the Preissman slot pipe method to study the transition from free surface flow to a surcharged state in the pipe (Cunge *et al.* 1980; Garcia Navarro *et al.* 1994; Capart *et al.* 1997; Ji 1998; Trajkovic *et al.* 1999; Ferreri *et al.* 2010). Other approaches are exemplified by a number of authors. They may be classified as rigid column method (Wiggert 1972; Hamam & McCorquodale 1982; Li & McCorquodale 1999; Zhou *et al.* 2002; Vasconcelos & Wright 2003), or full-dynamic models (Song *et al.* 1983; Cardle & Song 1988; Guo & Song 1990). Others preferred experimental methods, such as Jose & Steven (2005), and Ciraolo & Ferreri (2007), preferred experimental methods.

In this study, based on the Manning model and without taking into consideration the rapid filling of the pipe, a new approach is being proposed. It is much simpler and more accurate than the other methods for the computation of the flow velocity in partially filled pipes arranged in parallel form, for all the range of surface water angle: $0^\circ \leq \theta \leq 360^\circ$.

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THEORY

Manning equation

For a long time, the Manning equation (Manning 1891), has been frequently considered as the best formula to compute free surface flow owing to its simplicity. The Manning equation can be used for uniform flow. Graphs and tables are established to facilitate the application of the Manning equation for the estimation of flow characteristics (Camp 1946; Swarna & Modak 1990; McGhee 1991). Manning's equation can be written as follows:

$$Q = \frac{1}{n} R_h^{2/3} A S^{1/2} \tag{1}$$

$$V = \frac{1}{n} R_h^{2/3} S^{1/2} \tag{2}$$

where Q is the flow rate (m^3/s), $R_h^{1/2}$ is the hydraulic radius (m), n is the pipe roughness coefficient, or Manning n ($s/m^{1/3}$), A is the cross-sectional flow area (m^2), S is the slope of pipe bottom, dimensionless, and V is the flow velocity (m/s).

To use the Manning model some hypotheses must be assumed: the flow must be steady and uniform, where the slope, cross-sectional flow area, velocity are not related to time, and constant throughout the length of pipe being analysed (Carlier 1980).

Equations (1) and (2) can be rewritten as a function of the water surface angle of the pipe, as shown in Figure 1, as follows:

$$Q = \frac{1}{n}\left(\frac{D^8}{2^{13}}\right)^{1/3}\left[\frac{(\theta - \sin\theta)^5}{\theta^2}\right]^{1/3} s^{1/2} \tag{3}$$

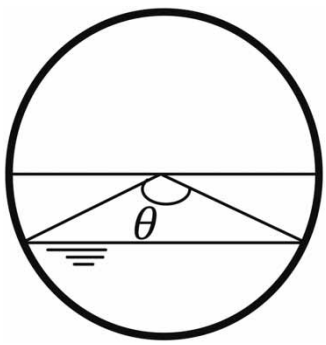

Figure 1 | Water surface angle.

$$V = \frac{1}{n}\left(\frac{D}{4}\right)^{2/3}\left[\frac{(\theta - \sin\theta)}{\theta}\right]^{2/3} s^{1/2} \tag{4}$$

$$A = \frac{D^2}{8}(\theta - \sin(\theta)) \tag{5}$$

$$P = \theta\frac{D}{2} \tag{6}$$

$$R_h = \frac{A}{P} = \frac{D}{4}\left(1 - \frac{\sin(\theta)}{\theta}\right) \tag{7}$$

where D is the pipe diameter (m), P is the wetted perimeter (m), and θ is the water surface angle (radian).

According to the equations mentioned above, the computation of the flow velocity is not direct, and needs to go through iterations. Several authors have attempted to propose approximate approaches to eliminate the need for trial procedures. Among these, we note those of Saatçi (1990), Giroud *et al.* (2000), and Akgiray (2005). Saatçi's approach is based on two steps. In the first, we should estimate the water surface angle using the following formula:

$$\theta_{\text{Saatçi}} = \frac{3\pi}{2}\sqrt{1 - \sqrt{1 - \sqrt{\frac{\pi Q n}{D^{8/3} S^{0.5}}}}} \tag{8}$$

In the second step, Equation (4) is used to calculate the flow velocity. The equations of Saatçi cannot be used for all the range of θ. They can be used only for the range: $\theta \leq 265^\circ$ (Saatçi 1990).

Giroud *et al.* (2000) and Akgiray (2005) have also proposed approaches in order to improve Saatçi's equation. The Giroud model proposes a direct relationship between average flow velocity and the flow rate for θ between 0° and 301.41°. This approach produces an estimate with a maximum deviation of less than 3%.

As well, Akgiray tried to improve the approaches presented earlier. He proposed approximate solutions from Manning's equation for four types of problems:

1. Given Q, D, and S, find h/D and/or V.
2. Given Q, D, and V, find h/D and/or S.
3. Given V, D, and S, find h/D and/or Q.
4. Given Q, V, and S, find h/D and/or D.

The four problems mentioned above are studied for two cases, when the Manning coefficient n is constant, and when n varies with the flow depth, as documented by Camp

(1946). In the latter, both variables are considered to be known.

In this research, we are interested in the first problem, for which Akgiray (2005) proposed the following equations to assess the water surface angle:

$$\theta = 2\times 6^{5/13}K^{3/13}\left(1+\left(\sin^{-1}(2.98K)\right)^{0.8}-2K^{0.946}\right) \quad (9)$$

where

$$K=\frac{Qn}{D^{8/3}S^{0.5}} \quad (10)$$

Equation (9) is proposed for θ between 0° and 301.41° with maximum error equal to 0.72%. The flow velocity can be estimated according to Equations (9) and (4), where the maximum deviation is 0.29%.

THE NEW APPROACH

Flow velocity

Subwatershed arranged in parallel

Subwatersheds may be arranged in series or in parallel. This study focuses on the second case, where the pipes are arranged in parallel as shown in Figure 2.

- The pipe M1-N1 collects water from subwatershed CC1 (which takes on number 1).
- The pipe M2-N1 collects water from the equivalent watershed CC2 (which takes on number 2).

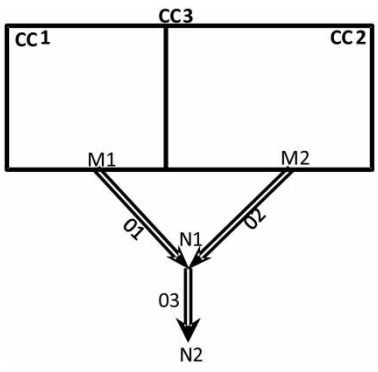

Figure 2 | Subwatershed and pipes arranged in parallel.

- The pipe N1-N2 collects water from the equivalent watershed CC3 (which takes on number 3).

$$CC_3 = CC_2 + CC_1 \quad (11)$$

The flow Q can be evaluated using current methods, such as the rational method or runoff curve number method (Viessman & Lewis 2003). Following are the four possible types of problems to compute the flow velocity in relation to the reference pipe:

1. The computation of V_3 as a function of the pipe 01 characteristics (pipe 01 is the reference pipe).
2. The computation of V_3 as a function of the pipe 02 characteristics (pipe 02 is the reference pipe).
3. The computation of V_2 as a function of the pipe 01 characteristics (pipe 01 is the reference pipe).
4. The computation of V_1 as a function of the pipe 02 characteristics (pipe 02 is the reference pipe).

Cases 1 and 2

Flow in pipes is steady and uniform, which means that the flow characteristics are constant in time and in space (throughout the length of pipe being analysed).

Let us consider that the pipe M2-N1 (or pipe 02) is the reference pipe, with known parameters. As such, the diameter D_2, hydraulic radius R_{h2}, surface water angle θ_2, cross-sectional water A_2 and slope S_2, are known data. The slope S_3 and roughness n_3 are considered as known parameters for the pipe N1-N2 (or pipe 03).

Equation (4) can also be written as the following equation (Zeghadnia *et al.* 2009):

$$V=\left(\left(\frac{S^{1/2}}{n}\right)^3\left(\frac{2Q}{D}\right)^2\right)^{1/5}\theta^{-2/5} \quad (12)$$

where θ is the water surface angle in radians as shown in Figure 2.

Q_1 is transported in pipe M1-N1 and produced in subwatershed CC1, Q_2 is transported in pipe M2-N1 and produced in subwatershed CC2, and Q_3 is transported in pipe N1-N2 and produced in subwatershed CC3.

$$Q_3 > Q_1 \quad (13)$$

$$Q_3 > Q_2 \quad (14)$$

The ratios Rq_{32} between Q_3 and Q_2 and Rq_{31} between Q_3 and Q_1, are given by the following equations:

$$\frac{Q_3}{Q_2} = Rq_{32} \tag{15}$$

$$\frac{Q_3}{Q_1} = Rq_{31} \tag{16}$$

where

$$Q_3 = A_3 V_3 \tag{17}$$

$$Q_1 = A_1 V_1 \tag{18}$$

From the inequalities shown in Equations (13) and (14) for just full pipe (under atmospheric pressure), we obtain the following equation:

$$A_3 = bA_1 \tag{19}$$

This gives:

$$D_3^2 = bD_1^2 \tag{20}$$

Similarly,

$$A_3 = aA_2 \tag{21}$$

where

$$D_3^2 = aD_2^2 \tag{22}$$

Based on Equations (15), (17), and (21), the ratio Rq_{32} becomes as follows:

$$Rq_{32} = \frac{aV_3}{V_2} \tag{23}$$

which yields:

$$V_3 = \frac{Rq_{32}}{a} V_2 \tag{24}$$

Using Equation (22), (2) becomes as follows:

$$V_3 = \frac{s_3^{0.5}}{n_3}\left(\frac{D_2}{4}\right)^{2/3} a^{1/3} \tag{25}$$

$$\Rightarrow a = V_3^3 \left(\frac{n_3}{s_3^{0.5}}\right)^3 \left(\frac{4}{D_2}\right)^2 \tag{26}$$

The combination of Equations (24) and (26) gives the following equation:

$$V_3 = (rQ_{32}V_2)^{1/4}\left(\frac{S_3^{0.5}}{n_3}\right)^{3/4}\left(\frac{D_2}{4}\right)^{1/2} \tag{27}$$

Equation (27) allows the computation of the flow velocity in just full pipe (pipe N1-N2) as a function of the reference pipe characteristics. In the case of partially filled pipe and according to Equation (12), the previous formula can be rewritten as follows:

$$V_3 = \left(\frac{Q_3}{Q_2}\right)^{1/4}\left(\frac{S_3^{0.5}}{n_3}\right)^{3/4}\left(\frac{n_2}{S_2^{0.5}}\right)^{3/20}\left(\frac{2Q_2}{D_2\theta_2}\right)^{2/5} \tag{28}$$

Similarly, in the case where pipe M1-N1 is the reference pipe, the flow velocity can be calculated as follows:

$$V_3 = \left(\frac{Q_3}{Q_1}\right)^{1/4}\left(\frac{S_3^{0.5}}{n_3}\right)^{3/4}\left(\frac{n_1}{S_1^{0.5}}\right)^{3/20}\left(\frac{2Q_1}{D_1\theta_1}\right)^{2/5} \tag{29}$$

Equations (28) and (29) give the exact value of flow velocity in pipe N1-N2 as a function of the reference pipe parameters, both for the first and for the second case, where the maximum deviation is zero as compared to Equation (4) results.

Cases 3 and 4

In these cases, the reference pipe characteristics are known. Here, we will write the characteristics of the first pipe using the characteristics of the second pipe (reference pipe) as will be shown in the following sections. Using Equations (20) and (22), we obtain the following equations:

$$aD_2^2 = bD_1^2 \tag{30}$$

$$D_1 = \left(\frac{a}{b}\right)^{0.5} D_2 \tag{31}$$

Using Equation (31), the velocity equation can be written as follows:

$$V_1 = \frac{s_1^{0.5}}{n_1}\left(\frac{D_2}{4}\right)^{2/3}\left(\frac{a}{b}\right)^{1/3} \tag{32}$$

Based on Equations (16), (17), and (19), the ratio Rq_{31} becomes as follows:

$$Rq_{31} = \frac{bV_3}{V_1} \tag{33}$$

which yields:

$$V_3 = \frac{Rq_{31}}{b} V_1 \tag{34}$$

Also, using Equations (24) and (34), we can get the following equation:

$$\frac{a}{b} = \frac{V_2 Q_1}{V_1 Q_2} \tag{35}$$

where

$$Rq_{12} = \frac{Q_1}{Q_2} \tag{36}$$

Table 1 | Accuracy test of Equation (38) compared to the recent approaches and Equation (4)

θ_1 and θ_2	Manning equation (4)	Proposed equation (38)	Error %	Saatçi equation error %	Giroud equation error %	Akgiray equation error %
1°	0.002175	0.002175	0	89.34740	0.004	3.82208×10^{-3}
2°	0.005480	0.005480	0	85.94367	0.004	1.83529×10^{-3}
3°	0.009410	0.009410	9.896×10^{-6}	83.46836	0.005	1.30634×10^{-3}
4°	0.013808	0.013808	0	81.45177	0.007	3.97250×10^{-3}
5°	0.018592	0.018592	0	79.71937	0.009	5.74054×10^{-3}
6°	0.023705	0.023705	0	78.18411	0.012	8.23439×10^{-3}
7°	0.029111	0.029111	6.398×10^{-6}	76.79537	0.015	1.16579×10^{-2}
8°	0.034779	0.034779	0	75.52073	0.018	1.50708×10^{-2}
9°	0.040686	0.040686	0	74.33798	0.022	1.90265×10^{-2}
10°	0.046813	0.046813	0	73.23115	0.027	2.34751×10^{-2}
20°	0.117604	0.117604	0	64.62856	0.096	9.48772×10^{-2}
35°	0.245945	0.245945	0	55.56500	0.276	0.295587
45°	0.341063	0.341063	0	50.64856	0.437	0.493892
90°	0.807894	0.807894	0	32.40590	1.190	2.001798
100°	0.911895	0.911895	0	28.60162	1.272	2.455752
120°	1.111704	1.111704	0	20.80760	1.244	3.462290
135°	1.250781	1.250781	0	14.60113	1.027	4.285882
145°	1.336877	1.336877	0	10.19018	0.791	4.857391
190°	1.641918	1.641918	0	14.59881	0.836	7.359813
200°	1.689098	1.689098	0	21.95952	1.192	7.810522
235°	1.792165	1.792165	6.651×10^{-6}	60.96910	1.749	8.712387
245°	1.804408	1.804408	0	80.98525	1.597	8.748280
256°	1.809674	1.809674	6.587×10^{-6}	119.68460	1.220	8.683651
290°	1.778588	1.778588	0	Not applicable	1.468	8.126489
300°	1.758345	1.758345	0	Not applicable	2.694	8.254137
308°	1.739430	1.739430	0	Not applicable	3.803	9.677319
335°	1.663581	1.663581	0	Not applicable	8.209	17.299310
345°	1.633169	1.633169	0	Not applicable	10.013	20.004720
360°	1.588002	1.588002	0	Not applicable	12.768	23.802130

Using Equations (32), (35), and (36), we may estimate the velocity using the characteristics of the second pipe for just full pipe as follows:

$$V_1 = (Rq_{12}V_2)^{1/4}\left(\frac{S_1^{0.5}}{n_1}\right)^{3/4}\left(\frac{D_2}{4}\right)^{1/2} \quad (37)$$

For the case of partially full pipe, and according to Equation (12), Equation (37) becomes:

$$V_1 = \left(\frac{Q_1}{Q_2}\right)^{1/4}\left(\frac{S_1^{0.5}}{n_1}\right)^{3/4}\left(\frac{n_2}{S_2^{0.5}}\right)^{3/20}\left(\frac{2Q_2}{D_2\theta_2}\right)^{2/5} \quad (38)$$

Equation (38) is the best formula as compared to existing methods with regard to Equation (4). This equation produces a maximum error of zero as shown in Table 1. For the case when pipe 01 is the reference pipe, the same result can be obtained as follows:

$$V_2 = \left(\frac{Q_2}{Q_1}\right)^{1/4}\left(\frac{S_2^{0.5}}{n_2}\right)^{3/4}\left(\frac{n_1}{S_1^{0.5}}\right)^{3/20}\left(\frac{2Q_1}{D_1\theta_1}\right)^{2/5} \quad (39)$$

Accuracy test

Table 1 shows that Equation (38) is an excellent formula, with a maximum deviation of $9.89 \times 10^{-6}\% \cong 0$, as compared to the Manning equation (Equation (4)) results. Existing methods, such as those of Saatçi (1990), Giroud *et al.* (2000), and Akgiray (2005), produce large error with limited range of application. For example, maximum deviation $\Delta V/V$ for Saatçi's equation is much higher and is not applicable for the entire range of θ. As well, the Giroud equation results for θ between 0° and 360° produce a maximum error of 12.76%. For the Akgiray formula, the maximum deviation is 23.80%. On the other hand, the proposed approach in this study using Equation (38) for the first case or Equation (39) for the second case is much more accurate, with a maximum error of zero. So far, it is the best existing formula as compared to Equation (4).

CONCLUSIONS

This paper presents a novel approach for the explicit computation of the flow velocity in partially filled circular-sections for four possible types of problems using known characteristics of a reference pipe. The proposed solution may be used for full circular sections. Its results are much better than those of existing models, where the deviation between the proposed equation and Manning results is zero as shown in Table 1. The proposed equation should be of great importance and application for concerned engineers and professionals.

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REFERENCES

Akgiray, Ö. 2004 Simple formula for velocity, depth of flow and slope calculations in partially filled circular pipes. *Environ. Eng. Sci.* **21** (3), 371–385.

Akgiray, Ö. 2005 Explicit solutions of the Manning equation in partially filled circular pipes. *Environ. Eng. Sci.* **32**, 490–499.

Camp, T. R. 1946 Design of sewers to facilitate flow. *J. Sewage Works* **18** (1), 3–16.

Capart, H., Sillen, X. & Zech, Y. 1997 Numerical and experimental water transients in sewer pipes. *J. Hydraul. Res.* **35** (5), 659–670.

Cardle, J. A. & Song, C. S. S. 1988 Mathematical modeling of unsteady flow in storm sewers. *Int. J. Eng. Fluid Mech.* **1** (4), 495–518.

Carlier, M. 1980 *Hydraulique Générale et Appliquée*. Eyrolles, Paris.

Chow, V.-T. 1959 *Open Channel Hydraulics*. McGraw-Hill, New York.

Ciraolo, G. & Ferreri, G. B. 2007 Experimental investigation on pressurization transient of a drainage sewer. In: *Proceedings of 5th International Symposium on Environmental Hydraulics – ISEH-V*, Tempe, Arizona (USA), December, on CD-ROM.

Cunge, J. A., Jr., Holly, F. M. & Verwey, A. 1980 *Practical Aspects of Computational River Hydraulics*. Pitman, London.

Ferreri, G. B., Freni, G. & Tomaselli, P. 2010 Ability of Preissmann slot scheme to simulate smooth pressurisation transient in sewers. *Water Sci. Technol.* **62** (8), 1848–1858.

Garcia-Navarro, P., Priestley, A. & Alcrudo, F. 1994 Implicit method for water flow modeling in channels and pipes. *J. Hydraul. Res.* **32** (5), 721–742.

Giroud, J. P., Palmer, B. & Dove, J. E. 2000 Calculation of flow velocity in pipes as function of flow rate. *J. Geosynth. Int.* **7** (4–6), 583–600.

Guo, Q. & Song, C. S. S. 1990 Surging in urban storm drainage systems. *J. Hydraul. Eng.* **116** (12), 1523–1537.

Hager, W. H. 2010 *Wastewater Hydraulics Theory and Practice*, 2nd edn. Springer, London.

Hamam, M. A. & McCorquodale, J. A. 1982 Transient conditions in the transition from gravity to surcharged sewer flow. *Can. J. Civ. Eng.* **9**, 189–196.

Henderson, F. M. 1966 *Open Channel Flow*. Macmillan, NewYork.

Ji, Z. 1998 General hydrodynamic model for sewer/channel network systems. *J. Hydraul. Eng.* **124** (3), 307–315.

Jose, G. V. & Steven, J. W. 2005 Experimental investigation of surges in a stormwater storage tunnel. *J. Hydraul. Eng.* **131** (10), 853–861.

Li, J. & McCorquodale, A. 1999 Modeling mixed flow in storm sewers. *J. Hydraul. Eng.* **125** (11), 1170–1180.

Manning, R. 1891 On the flow of water in open channels and pipes. *Trans. Inst. Civ. Eng. Ireland* **20**, 161–207.

McGhee, T. J. 1991 *Water Supply and Sewerage*, 6th edn. McGraw-Hill, New York.

Metcalf & Eddy Inc. 1981 *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, New York.

Saatçi, A. 1990 Velocity and depth of flow calculations in partially filled pipes. *J.* Environ. Eng. **116** (6), 1202–1208.

Song, C. S. S., Cardle, J. A. & Leung, K. S. 1983 Transient mixed flow models for storm sewers. *J. Hydraul. Eng.* **109** (11), 1487–1504.

Swamee, P. K. 1994 Normal depth equations for irrigation canals. *ASCE J. Hydraul. Div.* **102** (5), 657–664.

Swamee, P. K. & Rathie, P. N. 2004 Exact solution for normal depth problem. *J. Hydraul. Res.* **42** (5), 657–664.

Swarna, V. & Modak, P. 1990 Graphs for hydraulic design of sanitary sewers. *J. Environ. Eng.* **116** (3), 561–574.

Trajkovic, B., Ivetic, M., Calomino, F. & D'Ippolito, A. 1999 Investigation of transition from free surface to pressurized flow in a circular pipe. *Water Sci. Technol.* **39** (9), 105–112.

Vasconcelos, J. G. & Wright, S. J. 2003 Surges associated with expulsion in near-horizontal pipelines. In: *Proc. FEDSM03. 4th ASME-JSME Joint Fluids Engineering Conf.*, Honolulu, USA.

Viessman, W. & Lewis, G. L. 2003 *Introduction to Hydrology*, 5th edn. Prentice-Hall, Upper Saddle River, NJ, USA.

Wiggert, D. C. 1972 Transient flow in free-surface, pressurized systems. *J. Hydraul. Div. Am. Soc. Civ. Eng.* **98** (1), 11–27.

Zeghadnia, L., Djemili, L., Houichi, L. & Rezgui, N. 2009 Détermination de la vitesse et la hauteur normale dans une conduite partiellement remplie (Estimation of the flow velocity and normal depth in partially filled pipe). *J. EJSR.* **37** (4), 561–566.

Zhou, F., Hicks, F. E. & Steffler, P. M. 2002 Transient flow in a rapidly filling horizontal pipe containing trapped air. *J. Hydraul. Eng.* **128** (6), 625–634.

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