Analytic Solution for The computation of Flow Velocity and Water Surface Angle for drainage and Sewer networks: Case of Pipes arranged in series

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Abstract: Drainage and sewer network runs mostly under free surface flow condition. Among the characteristics which are important for practitioner are the flow velocity and water surface angle. The computation of these parameters in partially full pipes using Manning equation is implicit and requires iterative and laborious calculation methods. The goal of this paper is to provide a new method, where the exact computation of the flow velocity and water surface angle in partially filled pipe becomes easy, direct and simple using a reference pipe with known characteristics.

Key words: Water surface angle, free surface flow, uniform and steady flow, Circular pipe, Manning equation.

Biographical notes: Zeghadnia Lotfi was born in Souk Ahras,Algeria , on the 13th of April 1979. He obtained his graduate as Hydraulic engineering and his Magister from Badji Mokhtar university (in Annaba city, Algeria) in 2003 and 2007 respectively. He prepare his Phd in the same university. He is assistant lecturer at the University of Mohamed Cherif Messaadia Souk Ahras, Algeria.

1 INTRODUCTION

Drainage and sewer networks run mostly under free surface flow condition. If the flow is assumed to be uniform and steady, its characteristics can be estimated using Manning model, which is discussed by many authors such as Chow (1959), Henderson (1966), Metcalf & Eddy (1981), Carlier (1985), Hager (2010), among several others. Most drainage and sewer systems are built using circular shape conduits. During the conception phase, pipes can be arranged in series or in parallel form. Furthermore, the pipes can be flowing in just full (under atmospheric pressure) or in partially full section. For the first case, Manning equation is simpler and allow easy computation of pipe flow parameters (Hager, 2010). However, for the second case, which is the most frequently encountered, the Manning equations become more laborious and the iterative solution methods became necessary. Among these characteristics are the flow velocity and the water surface angle, which are important for flow control and measurement in pipes. Many authors have been trying to solve this problem by producing explicit solution for normal depth using Manning equation or Colebrook-white formula. These include Barr and Das (1986), wheeler (1992), Esen (1993), Swamee and al (2004), and Achour and Bdjaoui (2006).

Other authors such as Saâtçi (1990), Giroud and al (2000), and Akgray (2004 and 2005) have developed approximate methods for the case where the water surface angle ranges from 0 to 302.41°.

The goal of this study is to propose a new approach, which is much simpler and more accurate than existing methods for the computation of flow velocity and water surface angle for partially filled pipes arranged in series for the full range of surface water angle; ie, 0° ≤ θ ≤ 360°.

2 FLOW VELOCITY

Subwatershed arranged in series:

Subwatersheds (Sub) can be arranged in series as shown in Figure 1, or in parallel form. Our study will focus on the first case, where the pipes are arranged in series.

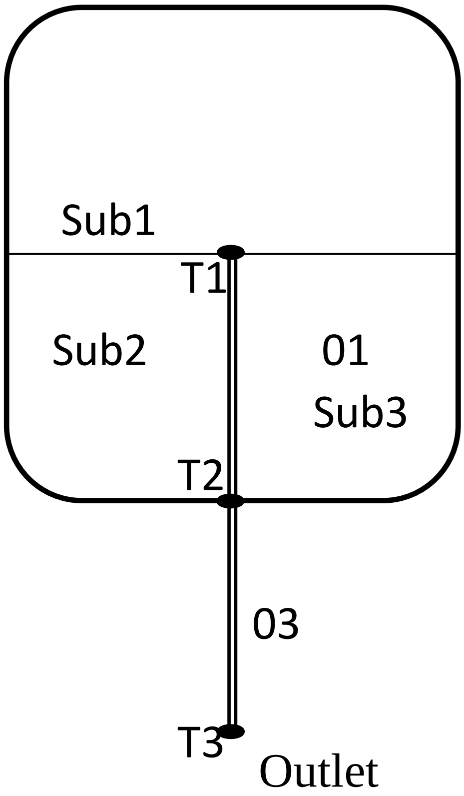

Figure 1: Subwatersheds and pipes arranged in series

The pipe T1-T2 collects water from subwatershed Sub1 (which takes on number 1) and T2-T3 collects water from the equivalent watershed Sub3 (which takes number 3), where:

$$Sub_{3=}\ Sub_2 + Sub_1 \qquad (1)$$

The flows can be estimated using existing methods such as the rational or the SCS (Viessman and Lewis, 2003). The flow in pipes is assumed to be steady and uniform, which means the flow characteristics are constant in space and time (along the length of the pipe being analyzed).

Let us to consider that pipe T1-T2 is the reference pipe, with known parameters, where the diameter D1, hydraulic radius Rh1, surface water angle θ1, water cross section A1, slope S1 are known. The slop S3 and roughness n3 are considered as known parameters for the pipe T2-T3.

Many Authors consider that Manning equation (Manning 1889) is the best model to describe free surface flow (Chow, 1959; Saatçi, 1990; Carlier, 1985; Akgray, 2004 and 2005; and Prabhata and Swamee, 1994 and 2004). Manning equation can be expressed as follow:

$$Q = \frac{1}{n} R_h^{2/3} A\, S^{1/2} \qquad (2)$$

$$V = \frac{1}{n} R_h^{2/3} S^{1/2} \qquad (3)$$

Where:

Q: flow rate, m3/s;

V : Velocity of flow, m/s;

A : cross section, m2

S: slope of pipe bottom, dimensionless

n: channel roughness coefficient (Manning n) , dimensionless

Rh: Hydraulic radius of channel, m.

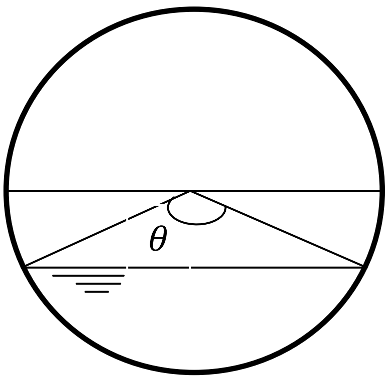

Figure 2: Water surface angle

From Figure 2 we can deduce the following:

$$Q = \frac{1}{n}\left(\frac{D^8}{2^{13}}\right)^{1/3}\left[\frac{(\theta - \sin\theta)^5}{\theta^2}\right]^{1/3} s^{1/2} \qquad (4)$$

$$V = \frac{1}{n}\left(\frac{D}{4}\right)^{2/3}\left[\frac{(\theta - \sin\theta)}{\theta}\right]^{2/3} s^{1/2} \qquad (5)$$

Equation (4) can be also written in the following form (Zghadnia et al, 2009):

$$V = \left(\left(\frac{S^{1/2}}{n}\right)^3 \left(\frac{2Q}{D}\right)^2\right)^{1/5} \theta^{-2/5} \qquad (6)$$

Where:

θ : Water surface angle in radians as shown in Figure 2.

Q_1 is transported in pipe T1-T2 and produced in subwatershed Sub1, Q_2 produced in subwatershed Sub2, and Q_3 is transported in pipe T2-T3 and produced in subwatershed Sub3

From Figure 1 we can deduce that:

$$Q_3 > Q_1 \qquad (7)$$

Where

$$Q_3 = A_3 V_3 \qquad (8)$$

$$Q_1 = A_1 V_1 \qquad (9)$$

The ratio Rq31 of Q_3 to Q_1 is given by the following equation:

$$Rq_{31} = \frac{Q_3}{Q_1} \qquad (10)$$

From the inequality (7) and for just full pipe (under atmospheric pressure), we can obtain the following:

$$A_3 = aA_1 \qquad (11)$$

Which gives:

$$D_3^2 = aD_1^2 \qquad (12)$$

For just full Manning equation, the flow velocity can be written as follow:

$$V_3 = \frac{S_3^{0.5}}{n_3}\left(\frac{D_3}{4}\right)^{2/3} \qquad (13)$$

When we substitute equation (12) in equation (13) and according to equations (8), (9), (10) and (11) we obtain the following equation (14):

$$V_3 = (\mathrm{Rq}_{31}\mathrm{V}_1)^{1/4}\left(\frac{\mathrm{S}_3^{0.5}}{\mathrm{n}_3}\right)^{3/4}\left(\frac{\mathrm{D}_1}{4}\right)^{1/2} \quad (14)$$

Equation (14) expresses the flow velocity in pipe T2-T3 in the just full flow pipe case under atmospheric pressure as a function of the flow characteristics of the first pipe.

For the case of partially full pipe, and according to the equation (6), equation (14) becomes as follow:

$$V_3 = \left(\frac{\mathrm{Q}_3}{Q_1}\right)^{1/4}\left(\frac{\mathrm{S}_3^{0.5}}{\mathrm{n}_3}\right)^{3/4}\left(\frac{n_1}{S_1^{0.5}}\right)^{3/20}\left(\frac{2Q_1}{D_1\theta_1}\right)^{2/5} \quad (15)$$

Accuracy Test:

Table 1 presents a comparison between the results of Equation 15 with those of Saatci's and those of Giroud's. Table 1 show that Equation 15 results are better than the other equations. Equation 15 produce results with a maximum error of 1.12810^{-5} %. .

An approximate equation has been proposed by Saatçi (1990) to solve the problem by calculating an approximate value of θ as follow:

$$\theta_{Saatçi} = \frac{3\pi}{2}\sqrt{1-\sqrt{1-\sqrt{\frac{\pi Q n}{D^{8/3}S^{0.5}}}}} \quad (16)$$

Then, the flow velocity can be obtained by substitution the approximate value of $\theta_{\mathrm{Saatçi}}$ into either equation (5) or equation (6).

Another approximate formula has been proposed by Giroud et al (2000) in order to improve Saatçi solution. It calculates the flow velocity as a function of the flow rate using the following formula:

$$V = 0.7591\left(1-\frac{Qn}{2D^{8/3}S^{0.5}}\right)\left(\frac{Q^4S^{9/2}}{D^2n^9}\right)^{1/13} \quad (17)$$

Both Saatçi and Giroud et al equations lack accuracy and the validity range of θ does not cover the entire possible values. The maximum error for the entire range of θ for Giroud et al equation is 12.77%, while that of Saatçi is 561.40%, beside being applicable for $\theta < 266°$ only.

Table 01: Accuracy test of equation (15) and equation (5), compared to Saatci's and Giroud's equations.

θ and θ_1	Manning Equation (5) (m/s)	Proposed Equation (15) (m/s)	Error %	Saatçi's equation Error %	Giroud's equation Error %
1°	0,00236	0,00236	9,86E-6	89.3474	0,004
2°	0,00595	0,00595	0	85,9436	0,004
3°	0,01021	0,01021	0	83,4683	0,005
4°	0,01499	0,01499	0	81,4517	0,007
5°	0,02018	0,02018	0	79,7193	0,009
6°	0,02573	0,02573	7,24E-6	78,1841	0,012
7°	0,03160	0,03160	0	76,7953	0,015
8°	0,03776	0,03776	0	75,5207	0,018
9°	0,04417	0,04417	8,43E-6	74,3379	0,022
10°	0,05082	0,05082	0	73,2311	0,027
20°	0,12769	0,12769	0	64,6285	0,096
25°	0,17154	0,17154	0	61,2826	0,146
35°	0,26703	0,26703	0	55,565	0,276
45°	0,37031	0,37031	0	50,6485	0,437
90°	0,87718	0,87718	0	32,4059	1,190
110°	1,10041	1,10041	0	24,7522	1,293
120°	1,20705	1,20705	0	20,8076	1,244
135°	1,35805	1,35805	0	14,6011	1,027
145°	1,45153	1,45153	0	10,1901	0,791
190°	1,78274	1,78274	0	14,5988	0,836
200°	1,83396	1,83396	0	21,9595	1,192
235°	1,9458	1,94587	0	60,9691	1,749
245°	1,95916	1,95916	0	80,9852	1,597
246°	1,96006	1,96006	0	83,4985	1,572
290°	1,93113	1,93113	0	not applicable	1,468
300°	1,90915	1,90915	6,24E-6	not applicable	2,694
308°	1,88861	1,88861	0	not applicable	3,803
320°	1,85394	1,85394	0	not applicable	5,652
335°	1,80626	1,80626	0	not applicable	8,209
345°	1,77324	1,77324	0	not applicable	10,013
350°	1,75671	1,75671	0	not applicable	10,930
360°	1,7242	1,7242	0	not applicable	12,768

3 WATER SURFACE ANGLE

As explained in the above sections, the surface water angle is an important parameter to control the flow in pipes, and in flow measurement. The computation of this parameter is needed for trial and error procedures. To eliminate the need for iterative methods, we use the following development:

From equation (6) we can deduce θ as follow:

$$\theta_3 = \left(\left(\frac{S_3^{0.5}}{n_3}\right)^3 \left(\frac{2Q_3}{D_3}\right)^2\right)^{1/2} \frac{1}{V_3^{5/2}} \qquad (18)$$

Due to the unknown parameter D3, equation (18) cannot produce directly the value of water surface angle of pipe 3.

From equations (18) and (15), it is easy to conclude that the expression of the water surface angle of pipe 3 (T2-T3) as a function of the known characteristics of reference pipe is as follow:

$$\theta_3 = \left(\frac{n_3}{S_3^{0.5}}\right)^{3/8} \left(\frac{Q_1}{Q_3}\right)^{5/8} \left(\frac{2Q_3}{D_3}\right) \left(\frac{D_1\theta_1}{2Q_1}\right) \left(\frac{S_1^{0.5}}{n_1}\right)^{3/8} \qquad (19)$$

From equation (19) and for a known diameter of pipe T2-T3, the computation of water surface angle become direct and much simpler. Furthermore, it gives the exact values of the water surface angle in pipe T2-T3 as a function of the reference pipe (T1-T2) characteristics.

Accuracy test:

Equation (19) is more accurate formula until now, where the error is practically null (10^{-15}%). Table 2 shows the difference between the results of the proposed model and those of Saatçi equation (16).

Table 02; Comparison between equation (19) results and those of Saatci equation (16).

θ *and* θ_1	Manning Equation(4) In radian	Proposed Equation (19) In radian	Error %	Saatçi equation (16) Error %
1°	0,02	0,02	0,00	2,69E+04
2°	0,03	0,03	0,00	1,34E+04
3°	0,05	0,05	0,00	8,90E+03
4°	0,07	0,07	0,00	6,65E+03
5°	0,09	0,09	0,00	5,30E+03
6°	0,10	0,10	0,00	4,40E+03
7°	0,12	0,12	0,00	3,76E+03
8°	0,14	0,14	0,00	3,27E+03
9°	0,16	0,16	0,00	2,90E+03
10°	0,17	0,17	0,00	2,60E+03
20°	0,35	0,35	0,00	1,24E+03
25°	0,44	0,44	0,00	9,72E+02
35°	0,61	0,61	0,00	6,60E+02
45°	0,79	0,79	0,00	4,84E+02
90°	1,57	1,57	0,00	1,66E+02
110°	1,92	1,92	0,00	1,04E+02
120°	2,09	2,09	0,00	7,92E+01
135°	2,36	2,36	0,00	4,84E+01
145°	2,53	2,53	0,00	3,08E+01
190°	3,31	3,31	0,00	2,89E+01
200°	3,49	3,49	0,00	3,91E+01
235°	4,10	4,10	0,00	6,96E+01
245°	4,27	4,27	0,00	7,73E+01
246°	4,29	4,29	0,00	7,81E+01
290°	5,06	5,06	0,00	not applicable
300°	5,23	5,23	0,00	not applicable
308°	5,37	5,37	0,00	not applicable
320°	5,58	5,58	0,00	not applicable
335°	5,84	5,84	0,00	not applicable
345°	6,02	6,02	0,00	not applicable
350°	6,11	6,11	0,00	not applicable
360°	6,28	6,28	0,00	not applicable

4 CONCLUSIONS

We have presented in this paper an analytic solution for the computation of flow velocity and water surface angle using a reference pipe in the case of subwatersheds arranged in series. This approach is applied to partially full or just full pipes (under atmospheric pressure). We have been able to calculate the pipe characteristics (velocity and water surface angle) directly and without lengthy and laborious solution procedures. Its results are much better than those of existing models.

NOTATION

Q: flow rate in m3/s;

V : Velocity of flow in m/s;

A : cross section in m2

S: slope of pipe bottom, dimensionless

n: channel roughness coefficient (Manning n)

Rh: Hydraulic radius of channel in m.

θ : Water surface angle in radians.

Rq: The ratio of flows

Sub1: subwatershed number 01.

Sub2: subwatershed number 02.

Sub3: the equivalent watershed.

T1-T2 : The pipe which collects water from subwatershed.

T2-T3 : The pipe which collects water from the equivalent watershed.

Q1 : flow produced in subwatershed 01.

Q2 : flow produced in subwatershed 02.

Q3 : flow produced in the equivalent watershed .

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