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A comparative study of nonlinear circle criterion based observer and H∞ observer for induction motor drive

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ABSTRACT

This paper deals with a comparative study of circle criterion based nonlinear observer and H_∞ observer for induction motor (IM) drive. The advantage of the circle criterion approach for nonlinear observer design is that it directly handles the nonlinearities of the system with less restriction conditions in contrast of the other methods which attempt to eliminate them. However the H_∞ observer guaranteed the stability taking into account disturbance and noise attenuation. Linear matrix inequality (LMI) optimization approach is used to compute the gains matrices for the two observers. The simulation results show the superiority of H_∞ observer in the sense that it can achieve convergence to the true state, despite the nonlinearity of model and the presence of disturbance.

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1. INTRODUCTION

For a long time, the induction motor is considered as a principal workhorse in the industry due to its robustness, high reliability, relatively low cost, modest maintenance requirements and efficiency [1, 2]. However, induction motor is also known as a complex nonlinear system, in which time-varying parameters entail additional difficulties for machine control, conditions monitoring and fault diagnostic purposes [1]. The main problem with induction motor industrial applications is that only a few state variables of the machine are available for on-line measurements. This is due to technical and/or economical constraints.

In order to perform advanced control techniques, conditions monitoring and faults diagnosis there is a great need of a reliable and accurate estimation of the key unmeasurable state variables of the machine. In this context, the observer design theory seems to be an ideal solution. Over the last two decades, nonlinear observer design problem has received much attention in the literature. Several attempts have been made for particular classes of nonlinear systems. Existing approaches can be roughly classified as follows: Nonlinear state linearization approaches, high-gain observers, geometric algorithms, variable structure design procedures, and algebraic techniques [3-6].

The problem with sensorless induction motor industrial applications is which approach that provide the most accurate and reliable estimation of unmeasurable state variables of the machine system.

In this paper we focus our attention on the performance comparison of the so called circle criterion approach and H_∞ observer design for induction motor (IM) system. The advantage of the circle criterion approach is the direct handling the system nonlinearities by exploiting their properties. This approach is less restrictive compared to the other methods as linearization approach, high gain observer and variable structure design procedures witch attempt to eliminate them [5, 6]. The H_∞ observer is used to guaranties the robustness against nonlinearities assumed to be equivalent to some uncertainties due to internal and extrernal disturbances and measurement noises [7-12].

To make a comparative study between the circle criterion based observer and H_∞ observer design for induction motor derive, we use one of the standard model of an induction motor. This type of nonlinear model is generally used for performing nonlinear control strategies, conditions monitoring and faults diagnosis of electric induction machine systems.

The paper is organized as follows: In the second and third section we present the theory of nonlinear observer and the nonlinear observer based circle criterion respectively. The robust H_∞ observer is presented in the fourth section. In the fifth section we present the considered nonlinear induction motor model. Finlay, we present simulation results and comments. A conclusion ends the paper.

2. THE NONLINEAR LUENBERGER OBSERVER DESIGN

We recall that an observer is a dynamical system which uses the available input-output data to reconstruct the unmeasurable system state variables. It is a "soft sensor" that plays an important role not only in sensorless control techniques but also in conditions monitoring, fault diagnosis, predictive maintenance and fault tolerant control techniques [1, 2].

In this paper, we consider a class of nonlinear systems that can be decomposed in linear and nonlinear parts as:

$$\dot{x}(t) = Ax(t) + F(x(t), u(t)) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

Where $x(t)$ stand for the system state variable, $u(t)$ for system input and $y(t)$ for the system output. The function $F(x(t), u(t))$ represents the system nonlinearities. For this type of nonlinear system, a general nonlinear observer (Luenberger) expression is as follows:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + F(\hat{x}, u) + L(y(t) - \hat{y}(t)) \tag{3}$$

$$\hat{y}(t) = C\hat{x}(t) \tag{4}$$

Where $\hat{x}(t)$ stands for the estimated state. We assume that the nonlinear function $F(x(t), u(t))$ is locally Lipschitz with respect to the state variable $x(t)$.

$\forall x(t), \hat{x}(t),\ \exists \gamma \,/\, \|F(x(t),u(t)) - F(\hat{x}(t),u(t))\| \le \gamma \|x(t) - \hat{x}(t)\|$ With $\gamma > 0$ as the Lipschitz constant;

Theorem 1 [13]: Consider Lipschitz nonlinear system (1)-(2) along with the observer (3)-(4). The observer error dynamics are asymptotically stable with maximum admissible Lipschitz constant if there exist scalers $\varepsilon > 0$ and $\xi > 0$ and matrices $P > 0$ and G such that the following LMI optimization problem has a solution $\min(\xi)$.

$$\begin{bmatrix} (A - LC)^T P + P(A - LC) & P \\ P & -\dfrac{P}{2\gamma} \end{bmatrix} \le 0 \tag{5}$$

$$\begin{bmatrix} \dfrac{1}{2}\xi \quad I & P \\ P & \dfrac{1}{2}\xi \quad I \end{bmatrix} > 0 \tag{6}$$

With $\gamma^* \hat{=} \max(\gamma) = \xi^{-1}$

Proof: Defining the observer error as $e(t) = x(t) - \hat{x}(t)$, then the observer error dynamics are given by:

$$\dot{e}(t) = (A - LC)e + F(x(t), u(t)) - F(\hat{x}(t), u(t)) \quad (7)$$

In the following we use notations:
$F = F(x(t), u(t))$ and $\hat{F} = F(\hat{x}(t), u(t))$

The time derivative of the Lyapunov function $V = e^T Pe$ can be expressed as:

$$\dot{V}(t) = e^T[(A - LC)^T P + P(A - LC)]e + 2e^T P(F - \hat{F}) \quad (8)$$

By using the Lipschitz property we have:

$$\Rightarrow 2\left\| e^T P(F - \hat{F}) \right\| \leq 2\gamma \|e\|^T P \|e\| \quad (9)$$

Then equation (8) becomes:

$$\dot{V}(t) = e^T[(A - LC)^T P + P(A - LC) + 2\gamma e^T P]e \leq 0 \quad (10)$$

Those leads to:

$$(A - LC)^T P + P(A - LC) + 2\gamma PP^{-1}P \leq 0 \quad (11)$$

By using the Schur complement theorem we therefore obtain the following linear matrix inequality (LMI):

$$\begin{bmatrix} (A - LC)^T P + P(A - LC) & P \\ P & -\dfrac{P}{2\gamma} \end{bmatrix} \leq 0 \quad (12)$$

The Lipschitz constant can be computed as follows:
For any Lyapunov function $V = e^T Pe$ we have:

$$\lambda_{\min}(P)\|e\|^2 \leq e^T Pe \leq \lambda_{\max}(P)\|e\|^2 \quad (13)$$

$$2\gamma e^T Pe \leq 2\gamma \lambda_{\min}(P)\|e\|^2 \quad (14)$$

Let: $(A - LC)^T P + P(A - LC) = -Q$ (15)

Then the time derivative can be written as:

$$\dot{V}(t) = -e^T Qe + 2\gamma P\|e\|^2 \leq 0 \quad (16)$$

$$\dot{V}(t) = -\lambda_{\min}(Q)\|e\|^2 + 2\gamma\lambda_{\max}(P)\|e\|^2 \leq 0 \quad (17)$$

So: $2\gamma\lambda_{\max}(P) \leq \lambda_{\min}(Q)$

$$\Rightarrow \gamma \leq \frac{1}{2}\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} \quad (18)$$

In addition, since P is positive definite, suppose $Q = I$ .From the equation (18) we can write:

$$1-2\gamma\lambda_{\max}(P)>0 \tag{19}$$

And $\bar{\sigma}(P)=\lambda_{\max}(P)$

So from (19):

$$\bar{\sigma}(P)<\frac{1}{2\gamma} \tag{20}$$

Which is equivalent to:

$$(\frac{1}{2\gamma})^2 I-P^T P>0 \tag{21}$$

Using Schur complement lemma

$$\begin{bmatrix} \frac{1}{2\gamma}I & P \\ P & \frac{1}{2\gamma}I \end{bmatrix}>0 \tag{22}$$

Defining $\xi=\frac{1}{\gamma}$, (6) is achieved.

3. CIRCLE CRITERION BASED NONLINEAR OBSERVER DESIGN

In contrast of the linearization-based and high-gain approaches which attempt to eliminate the system nonlinearities using a nonlinear state transformation or to dominate them by a high gain term of correction, circle-criterion exploits the properties of the system nonlinearities. In its basic form, introduced by Arcak and Kokotovic [14], the approach is applicable to a class of systems that can be decomposed in linear and nonlinear parts with a condition that the nonlinearities satisfy the sector property [14-16].

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3.1. Basic sector properties

A memoryless nonlinear function $F(z,t):[0\ +\infty[\times R^p \to R^p$ is said to belong to the sector $[0\ +\infty[$ if $zF(z,t)\geq 0$. Let v_1 and v_2 two real positive numbers, by setting $z=v_1-v_2$ and $F(z,t)=[F(v_1,t)-F(v_2,t)]$, the above sector property is equivalent to:

$$(v_1-v_2)[F(v_1,t)-F(v_2,t)]\geq 0\ \forall v_1,v_2\in R^+ \tag{23}$$

Relation (23) states that the nonlinear function $F(z,t)$ is a nondecreasing function. On the other hand if $F(z,t)$ is a continuously differentiable function the above relation is equivalent to [7], [8]:

$$\frac{d}{dz}F(z,t)\geq 0 \quad \forall z\in R \tag{24}$$

If the nonlinear function $F(z,t)$ does not satisfy the positivity condition (24) we introduce a function $\bar{F}(z,t)$ such that:

$$\bar{F}(z,t)=F(z,t)+\rho z, \quad \rho\succ\left\|\frac{d}{dz}F(z,t)\right\|, \forall z\in R \tag{25}$$

And:

$$\frac{d}{dz}\overline{F}(z,t) = \frac{d}{dz}F(z,t) + \rho \geq 0 \quad \forall z \in R \tag{26}$$

In the multivariable case the sector property can be written as: $z^T F(z,t) \geq 0$. Where z and $F(z,t)$ are respectively vectors of an appropriate dimension.

3.2. Nonlinear Observer Design

The circle criterion based nonlinear observer design can be performed for a class of nonlinear system that the model can be decomposed into linear part and nonlinear part as the following [14-16]:

$$\dot{x}(t) = Ax(t) + \phi[u(t), y(t)] + GF[H.x(t)] \tag{27}$$

$$y(t) = Cx(t) \tag{28}$$

Where A , C and G are known constant matrices with appropriate dimensions. The pair (A,C) is assumed to be observable. The term $\phi[y(t),u(t)]$ is an arbitrary real-valued vector that depends only on the system measured control inputs $u(t)$ and outputs $y(t)$. The nonlinear part of the system is included in the term $F[H.x(t)]$ which is a time-varying vector function verifying the sector property. In the following we recall the main theorem and conditions that are used in this work to study the feasibility of nonlinear observer design for induction motor sensorless control with respect of circle criterion or sector property. A detailed proof of the theorem is presented in reference [17].

Theorem 2 [14, 15]: Consider a nonlinear system of the form (27)-(28) with the nonlinear part satisfying the circle criterion relations (23)-(26). If there exist a symmetric and positive definite matrix $P \in R^{nxn}$ and a set of row vectors $K \in R^p$ such that the following linear matrix inequalities (LMI) hold:

$$(A - LC)^T P + P(A - LC) + Q \leq 0 \tag{29}$$

$$PG + (H - KC)^T = 0 \tag{30}$$

The nonlinear observer design refers to the selection of the gain matrices L and K satisfying the LMI conditions (29)-(30). With $Q = \varepsilon I_n$ as a defined positive known matrix, I_n is an n-th order unity matrix and ε is a small positive real number.

Then a nonlinear observer can be designed as:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + \phi[u(t), y(t)] + L[y(t) - \hat{y}(t)] + GF[H\hat{x}(t) + K(y(t) - \hat{y}(t))] \tag{31}$$

$$\hat{y}(t) = C\hat{x}(t) \tag{32}$$

And $\lim_{t\to\infty} e(t) = x(t) - \hat{x}(t) = 0$, where $\hat{x}(t)$ is the estimate of the state vector $x(t)$ of the nonlinear system. One can see that the structure of the nonlinear observer is composed of a linear part that is similar to linear Luenberger observer and a nonlinear part that is an additional term that represents the time-varying nonlinearities satisfying the sector property.

Proof: The dynamics of the state estimation error $e(t) = x(t) - \hat{x}(t)$ are given by:

$$\dot{e}(t) = (A - LC)e(t) + G[f(H.x(t)) - F(H.\hat{x}(t) + K(y(t) - \hat{y}(t))] \tag{33}$$

Let $v_1 = H.x(t)$ and $v_2 = H.\hat{x}(t) + K(y(t) - \hat{y}(t))$, by setting $z = v_1 - v_2 = (H - KC)e(t)$, the term between brackets in (33) can be seen as a function of the variable z then: $[f(v_1) - f(v_2)] = f(z,t)$.
The expression $(v_1 - v_2)[F(v_1) - F(v_2)] = zF(z,t)$ satisfies the property of the sector.
Taking into account the above result, the error dynamics in (33) can be rewritten as:

$$\dot{e}(t) = (A - LC)e(t) + G.F(z,t) \tag{34}$$

$$z = (H - KC)e(t) \tag{35}$$

Relations (34)-(35) show, once again, that the error dynamics can then be considered as a linear system controlled by a time-varying nonlinearity function $F(z,t)$ satisfying the sector property. Circle criterion establishes that the feedback interconnection of a linear system and a time-varying nonlinearity satisfying the sector property is globally uniformly asymptotically stable [14, 15].

Based upon the error dynamics, relation (34)-(35), the nonlinear observer design problem is then equivalent to stabilization of the error dynamics problem. To this end a candidate Lyapunov function $V = e^T Pe$ is considered. In order to ensure asymptotic stability of the observer, the derivative of the candidate Lyapunov function must be negative. With the help of relation (34) and (35) the derivative of the Lyapunov function becomes:

$$\dot{V} = e^T\left[(A - LC)^T P + P(A - LC)\right]e + F^T(z,t)G^T Pe + e^T PGF(z,t) \tag{36}$$

By setting:

$$(A - LC)^T P + P(A - LC) \le -Q \tag{37}$$

And

$$PG = -(H - KC)^T \tag{38}$$

With $Q = \varepsilon I_n$ and $\varepsilon > 0$, the derivative of the Lyapunov function can be rewritten as:

$$\dot{V} \le -e^T Qe - 2.z^T.F(z,t) \tag{39}$$

Thus ends the proof.
Note that the existence of observer (31)-(32) is conditioned by the solution of LMI conditions (29)-(30). By solving LMI constraints, observer gain matrices L and K that guarantee observer convergence are then computed. In Ibrir [16], the author has investigated the study of globally Lipschitz systems and bounded-state nonlinear systems. Bounded-state nonlinear systems constitute a large class of system that includes electric machine systems. Electric machine models involve the magnetic flux as a key and bounded state variable that combined with other state variable of the machine, such as rotor angular velocity, leads to the nonlinear part of the machine model. This is due to the effect of the magnetic material saturation property that is similar to the sector nonlinearity.

4. NONLINEAR H_∞ OBSERVER SYNTHESIS

In this section, we propose to study the robust H_∞ observer. This observer is used in the case of systems involving uncertainties in the model measurements [7, 9, 10, 11, 12, 18].
A literature review shows that many useful H_∞ filtering approaches have been developed for several kinds of systems. One can cite the H_∞ observers for one-sided Lipschitz nonlinear systems, the H_∞ observer for singular Lipschitz nonlinear systems and the H_∞ control and filtering for uncertain Markovian jump systems with time-varying delay [9, 10, 11, 19].

The aim of the H_∞ approach is to design a full-order filter such that the corresponding filtering error system is asymptotically stable and satisfies a prescribed H_∞ level disturbance attenuation. By using a Lyapunov function, sufficient conditions are formulated in terms of linear matrix inequalities (LMIs).

To do this, we extend the result of the previous section to nonlinear robust $H\infty$ observer design method. We consider the system (1)-(2) with an additional disturbance term as follows:

$$\dot{x}(t) = Ax(t) + F(x(t), u(t)) + B_d w(t) \tag{40}$$

$$y(t) = Cx(t) \tag{41}$$

Where $w \in l_2[0,\infty)$ is an unknown exogenous disturbance.

The goal is to rebuild the state $x(t)$ of the system (40)-(41) with some accuracy despite the presence of disturbance term $w(t)$.

We suppose that: $z = He(t)$, with H a known constant matrix such that $\|z\| \prec \mu\|w\|$, with $\mu \succ 0$.

The corresponding observer for the system (40)-(41) is given by:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + F(\hat{x}(t), u(t)) + L[y(t) - \hat{y}(t)] \tag{42}$$

$$\hat{y}(t) = C\hat{x}(t) \tag{43}$$

Our purpose is to design the observer parameter ***L*** such that the observer error dynamics are asymptotically stable and the following specified norm H_∞ upper bound is simultaneously guaranteed.

Lemma 1: For any $x, y \in R^n$ and any positive definite matrix $P \in R^{n\times n}$, we have: $2x^T y \le x^T Px + y^T P^{-1} y$

Theorem 3 [13]: Consider stochastic Lipschitz nonlinear system (40)-(41), and the corresponding observer (42)-(43). The observer error dynamics are asymptotically stable with the minimum of the norm l_2 and μ, if there exist scalers $\alpha_s >$ 1, $\varepsilon > 0$ and $\xi > 0$ and matrices $P > 0$ and G such that the following LMI optimization problem has a solution.

$\min(\xi)$

$$\begin{bmatrix} H^T H + \frac{1}{2}(\gamma + \frac{1}{\gamma} - \alpha_s)I & PB_d \\ B_d^T P & -\xi I \end{bmatrix} < 0 \tag{44}$$

$$\begin{bmatrix} \frac{1-\bar{\sigma}^2(H)}{2\gamma} I & P \\ P & \frac{1-\bar{\sigma}^2(H)}{2\gamma} \end{bmatrix} < 0 \tag{45}$$

With $L = P^{-1}G$ and $\mu^* \hat{=} \min(\mu) = \sqrt{\xi}$

Proof: The observer error dynamics are:

$$\dot{e}(t) = (A - LC)e(t) + (F - \hat{F}) + B_d w(t) \tag{46}$$

The time derivative of the Lyapunov function $V(t) = e^T Pe(t)$ is given by:

$$\begin{aligned} \dot{V}(t) &= e^T[(A-LC)^T P + P(A-LC)]e \\ &+ 2e^T P(F-\hat{F}) + e^T PB_d w(t) + w(t)^T B_d^T Pe \end{aligned} \tag{47}$$

Using Lemma 1 and the Rayleigh leads to the following:

$$2e^T P(F-\hat{F}) \le e^T Pe + (F-\hat{F})^T PP^{-1}P(F-\hat{F}) \tag{48}$$

$$2e^T P(F-\hat{F}) \le e^T Pe + (F-\hat{F})^T P(F-\hat{F}) \tag{49}$$

The right term of (49) can be written as:

$$e^T Pe \le \lambda_{\max}(P)\|e\|^2 = \lambda_{\max}(P)e^T e \tag{50}$$

$$\left\|(F-\hat{F})P(F-\hat{F})\right\|^T \le \gamma^2\lambda_{\max}(P)\|e\|^2 \tag{51}$$

The left term of (49) can be written as:

$$2\left\|e^T P(F-\hat{F})\right\| \le 2\gamma\lambda_{\max}(P)\|e\|^2 \tag{52}$$

Replacing (51)-(52) in equation (49), leads to:

$$2\gamma\lambda_{\max}(P)\|e\|^2 \le \lambda_{\max}(P)\|e\|^2 + \gamma^2\lambda_{\max}(P)\|e\|^2$$

Assume $Q = I$, in the deterministic case we have $w = 0$ and the corresponding time derivative of the function Lyapunov becomes:

$$\dot{V}(t) \le -e^T Qe + 2e^T(F-\hat{F}) \le 0 \tag{53}$$

$$\dot{V}(t) \le -e^T Ie + 2\gamma\lambda_{\max}(P)\|e\|^2 \le 0 \tag{54}$$

Then:

$$1 - 2\gamma\lambda_{\max}(P) \ge 0 \tag{55}$$

$$\bar{\sigma}(P) \le \frac{1}{2\gamma} \tag{56}$$

$$2e^T P(F-\hat{F}) \le (1+\gamma^2)\lambda_{\max}(P)e^T e \tag{57}$$

$$\begin{aligned} 2e^T P(F-\hat{F}) &\le (1+\gamma^2)\frac{1}{2\gamma}(P)e^T e \\ &\le \frac{1}{2}(\gamma + \frac{1}{\gamma})e^T e \end{aligned} \tag{58}$$

In the stochastic and general case of $w \ne 0$ and $Q = \alpha_s I$ the above equation of Lyapunov derivative becomes:

$$\begin{aligned} \dot{V}(t) &= e^T[(A-LC)^T P + P(A-LC)]e \\ &+ 2e^T P(F-\hat{F}) + e^T PB_d w(t) + w(t)^T B_d^T Pe \end{aligned} \tag{59}$$

$$\begin{aligned} \dot{V}(t) &\le -\lambda_{\max}(Q)e^T e + \frac{1}{2}(\gamma + \frac{1}{\gamma})e^T e \\ &+ w^T(t)B_d^T Pe + e^T PB_d w(t) \end{aligned} \tag{60}$$

$$\dot{V}(t) = -\alpha_s e^T e + \frac{1}{2}(\gamma + \frac{1}{\gamma})e^T e + w^T(t)B_d^T Pe + e^T PB_d w(t) \tag{61}$$

$$\dot{V}(t) = [\frac{1}{2\gamma}(1+\gamma^2) - \alpha)]e^T e + w^T(t)B_d^T Pe + e^T PB_d w(t) \tag{62}$$

$z = He(t)$, H is a known constant with $\|z\| \prec \mu\|w\|$

Now, we define the following criterion to minimize the difference between the energy of the estimation error and the energy inducted by the disturbances:

$$J = \int_0^\infty (z^T z - \zeta w^T w)dt \tag{63}$$

Thus:

$$J < \int_0^\infty (z^T z - \zeta w^T w + \dot{V}(t))dt \tag{64}$$

So a sufficient condition for $J < 0$ is that:

$$\forall t \in [0, \infty), \quad z^T z - \zeta w^T w + \dot{V}(t) \prec 0$$

This means that: $z^T z - \zeta w^T w < 0$ (because $\dot{V}(t)$ is a negative function) hence:

$$\|z\|^2 < \zeta\|w\|^2 \quad \Rightarrow \quad \|z\| < \sqrt{\zeta}\|w\|.$$

But we have:

$$\begin{aligned} z^T z - \zeta w^T w + \dot{V}(t) &= e^T H^T He - \zeta w^T w + \dot{V}(t) \\ &\le e^T H^T He + [\frac{1}{2\gamma}(1+\gamma^2) - \alpha_s)]e^T e \\ &+ e^T PB_d w + w^T B_d^T Pe - \zeta w^T w \end{aligned} \tag{65}$$

Then, we can write:

$$J = \begin{bmatrix} e^T & w^T \end{bmatrix} \begin{bmatrix} H^T H + [\frac{1}{2\gamma}(1+\gamma^2) - \alpha_s]I & PB_d \\ B_d^T P & -\zeta I \end{bmatrix} \begin{bmatrix} e \\ w \end{bmatrix} \tag{66}$$

So a sufficient condition for $J < 0$ is that the following be negative definite:

$$\begin{bmatrix} H^T H + [\frac{1}{2\gamma}(1+\gamma^2) - \alpha_s]I & PB_d \\ B_d^T P & -\zeta I \end{bmatrix} < 0 \tag{67}$$

According to the Schur's complement lemma, (67) is equivalent to: $-\xi I < 0$.

$$H^T H + [2\gamma\lambda_{\max}(P) - \alpha_s]I + \frac{1}{\xi}PB_d B_d^T P < 0 \tag{68}$$

For equation (68) to be negative, the sum of the first and second terms must be negative because the third term is always positive:

$$H^T H + [2\gamma\lambda_{\max}(P) - \alpha_s]I < 0 \tag{69}$$

But as for any other symmetric matrix, for $H^T H$, we have:

$$\lambda_{\min}(H^T H) \le H^T H \le \lambda_{\max}(H^T H) \tag{70}$$

$$\sigma_{\max}^2(H)I \ge H^T H \ge \sigma_{\min}^2(H)I \tag{71}$$

Or according to the definition of singular values

$$\bar{\sigma}^2(H) + 2\gamma\lambda_{\max}(P) - \alpha_s < 0 \tag{72}$$

Or:

$$\lambda_{\max}(P) < \frac{\alpha_s - \bar{\sigma}^2(H)}{2\gamma} \tag{73}$$

This is equivalent to (45).

5. SIMULATION RESULTS AND COMMENTS

To perform the comparative study between the two nonlinear observers previously view, we use the model of an induction motor. Described by the following nonlinear differential equations with, the stator current, rotor flux and rotor angular velocity as selected state variables of the machine [1, 20, 21].

$$\begin{aligned}
\frac{d}{dt} i_{s\alpha} &= -\gamma\, i_{s\alpha} + \frac{\beta}{T_r}\varphi_{r\alpha} + \beta\,\omega_r\varphi_{r\beta} + \frac{1}{\sigma l_s}u_{s\alpha} \\
\frac{d}{dt} i_{s\beta} &= -\gamma\, i_{s\beta} - \beta\,\omega_r\varphi_{r\alpha} + \frac{\beta}{T_r}\varphi_{r\beta} + \frac{1}{\sigma l_s}u_{s\beta} \\
\frac{d}{dt}\varphi_{r\alpha} &= \frac{m}{T_r} i_{s\alpha} - \frac{1}{T_r}\varphi_{r\alpha} - \omega_r\varphi_{r\beta} \\
\frac{d}{dt}\varphi_{r\beta} &= \frac{m}{T_r} i_{s\beta} + \omega_r\varphi_{r\alpha} - \frac{1}{T_r}\varphi_{r\beta} \\
\frac{d}{dt}\omega_r &= \alpha(\varphi_{r\alpha} i_{s\beta} - \varphi_{r\beta} i_{s\alpha}) - k_f\omega_r - k_l T_l
\end{aligned} \tag{74}$$

Where: $\Omega_r = \frac{\omega_r}{n_p}$, $\alpha = \frac{n_p^2 m}{J l_r}$, $\beta = \frac{1}{m}\left(\frac{1-\sigma}{\sigma}\right) = \frac{1}{\sigma}\frac{m}{l_s l_r}$, $\sigma = 1 - \frac{m^2}{l_s l_r}$, $k_f = \frac{f_r}{J}$, $k_l = \frac{n_p}{J}$, $T_r = \frac{L_r}{R_r}$.

The indexes s and r refer to the stator and the rotor components respectively. i , φ and u respectively denote the stator current, the rotor fluxes, the supplied stator voltage, R is the resistance, l is the inductance, m is the mutual inductance. T_s and T_r are the stator and the rotor time constant respectively. ω_r is the rotor angular velocity, f_r is the friction coefficient, J is the moment of inertia coefficient, n_p is the number of pair poles, Ω_r is the mechanical speed of the rotor and finally T_l is the mechanical load torque.

This type of nonlinear model is generally used for performing nonlinear control, conditions monitoring and faults diagnosis of electric induction machine systems. Performing these techniques requires estimating unmeasured rotor flux linkage and rotor angular velocity state variables based on the stator current and voltage measurements [21].

To carry out the comparative study between the two observers previously view, we use the induction motor with the following characteristics details on Table 1.

Note that the starting conditions X0= [4 0 0 0 0] for the initial state vector of the machine are taken into account in simulations of the two observers.

In the application of the nonlinear observer based on the criterion of circle approach, we must write the model of induction motor (74) in the form (27)-(28), taking into account the properties (29) and (30).

Table 1 Characteristics of the induction motor

Symbol	Quantity	Numerical value
P	Power	1.5 KW
f	Supply frequency	50 Hz
U	Supply voltage	220 V
n_p	Number of pair poles	2
R_s	Stator resistance	4.850 Ω
R_r	Rotor resistance	3.805 Ω
l_s	Stator inductance	0.274 H
l_r	Rotor inductance	0.274 H
m	Mutual inductance	0.258 H
ω_r	Rotor angular speed	297.25 rd/s
J	Inertia coefficient	0.031 kg^2/s
f_r	Fiction coefficient	0.00114 N.s/rd
T_l	Load torque	5 N.m

The nonlinearities of the machine system are function of the flux state variable that is a bounded state variable. The nonlinearities of the model are of the form $\omega_r\varphi_{rd}$ that can be expressed as:

$$\omega_r\varphi_{rd} = (\omega_r\varphi_{rd} + \rho\omega_r) - \rho\omega_r \tag{75}$$

One can verify that:

$$\frac{\partial}{\partial\omega_r}(\omega_r\varphi_{rd} + \rho\omega_r) = \varphi_{rd} + \rho \geq 0 \tag{76}$$

With $\|\varphi_{rd}\| \leq 2$, then one can choose $\rho = 2$.

The first step of the simulation consists of resolving the LMI conditions, relation (29)-(30), using an adequate LMI tools such as the LMI tool-box of the Matlab software. The obtained nonlinear observer gain matrices L and K are the following:

$$L = \begin{bmatrix} -1.6749 & 0.1188 \\ 0.1188 & -1.6749 \\ -0.7172 & -0.1075 \\ -0.1075 & -0.7172 \\ 1.6201 & -1.6201 \end{bmatrix}$$

$K_1 = [-1.6037 \quad -0.7381]$, $K_2 = [0.7381 \quad 1.6037]$, $K_3 = [0.3948 \quad -0.9193]$, $K_4 = [-0.9193 \quad 0.3948]$

The corresponding Lyapunov matrix for this LMI feasibility test is:

$$P=\begin{bmatrix} 0.1550 & -0.0710 & 0.0514 & 0.1486 & 0.0274 \\ -0.0710 & 0.1550 & 0.1486 & 0.0514 & -0.0274 \\ 0.0514 & 0.1486 & 5.6010 & 0.4659 & -0.0505 \\ 0.1486 & 0.0514 & 0.4659 & 5.6010 & 0.0505 \\ 0.0274 & -0.0274 & -0.0505 & 0.0505 & 0.0173 \end{bmatrix}$$

With: $\varepsilon = 0.04$.

The second step of simulation consists of injecting the obtained numerical values of the gain matrix and the vectors in an S-function-based Matlab program that interacts with the Matlab Simulink software to simulate the nonlinear system and the nonlinear observer.

The simulation results of the designed nonlinear observer based circle criterion are presented in the following. Figure 1 and Figure 2 show the measured and estimated stator current and rotor flux components respectively. The Figure 3 and Figure 4 show the measured and estimated rotor angular velocity and the corresponding load torque respectively with corresponding estimation error. One can see that the estimated state variables of the machine follow the desired trajectories.

To highlight these results a load torque is introduced in the simulation at time of $0.5\,\sec$, the simulation results show that all the state variables of the machine are modified accordingly. Thus demonstrate the effectiveness of the circle criterion based nonlinear observer design for the induction machine system state estimation.

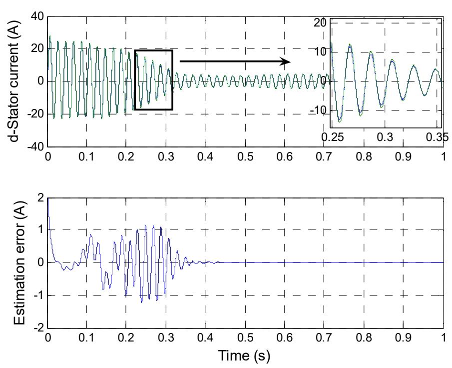

Figure 1. Measured (blue line) and observed(green line) *d*-stator current components and the corresponding estimation error

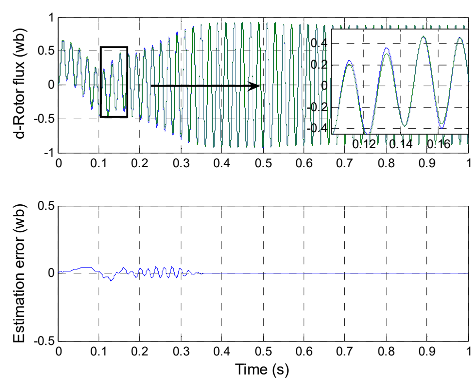

Figure 2. Measured (blue line) and observed (green line) *d*-rotor flux components and the corresponding estimation error

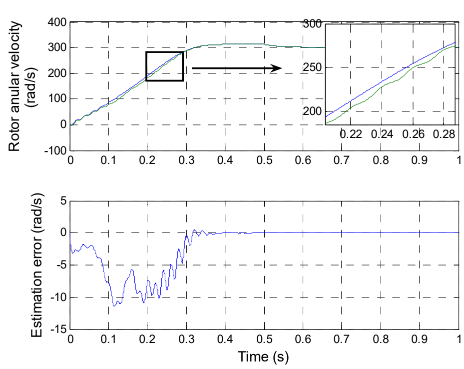

Figure 3. Measured (blue line) and observed (green line) rotor angular velocity of the (IM) and the corresponding estimation error

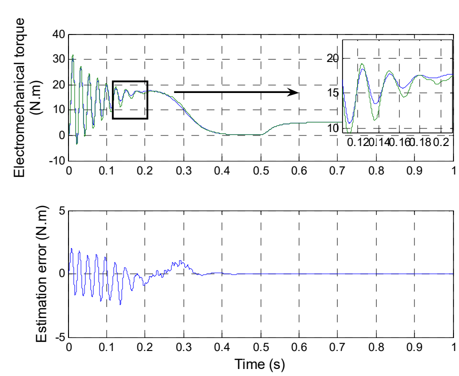

Figure 4. Measured (blue line) and estimated (green line) electromechanical torque of the (IM) and the corresponding estimation error

In simulation experiments of nonlinear H_∞ observer we must write the model of induction motor (74) in the form (40) - (41).

As in the first observer approach, the simulation of the nonlinear H_∞ approach is performed in two steps, the first simulation step consists of resolving the LMI conditions, relation (44)-(45), using an adequate LMI tools such as the LMI tool-box of the Matlab software.
To ensure robustness against nonlinear uncertainty, we use the theorem 1 to maximize the admissible Lipschitz constant γ and then the theorem 3, to minimize μ for the maximized γ .
$\gamma^* = 0{,}7$, $\mu^* = 0{,}112$, $\varepsilon = 0{,}04$.
Subsequently, from Theorem 3 and solving the LMI, (40), (41) and (42), then we calculate the nonlinear observer gain matrices L and G.
In this observer, we apply a disturbance in the power supply u in order to see these performances.
$H = 10I$, $\alpha_s = 0.05$, $Q = 5I_5$, $B_d = 20$

$$P=\begin{bmatrix} 0.0005 & 0 & 0.0143 & 0 & 0 \\ 0 & 0.0005 & 0 & 0.0143 & 0 \\ 0.0143 & 0. & 0.4552 & 0 & 0 \\ 0 & 0.0143 & 0. & 0.4552 & 0. \\ 0 & 0 & 0 & 0 & 1.6924 \end{bmatrix} \quad G=\begin{bmatrix} -0.0574 & 0 \\ 0 & -0.0574 \\ -1.7151 & 0 \\ 0 & -1.7151 \\ 0 & 0 \end{bmatrix} \quad L=\begin{bmatrix} -213.5922 & 0 \\ 0 & -213.5922 \\ 2.9415 & 0 \\ 0 & 2.9415 \\ 0 & 0 \end{bmatrix}$$

The second step of simulation consists of injecting the obtained numerical values of the gain matrix and the vectors in an S-function-based Matlab program that interacts with the Matlab Simulink software to simulate the nonlinear system and the nonlinear H_∞ observer.

The simulation results of the designed and nonlinear H_∞ observer are presented in the following. Figure 5 and Figure 6 show the measured and estimated stator current and rotor flux components respectively. The Figure 7 and Figure 8 show the measured and estimated rotor angular velocity and the corresponding load torque respectively with corresponding estimation error. One can see that the estimated state variables of the machine follow the desired trajectories.

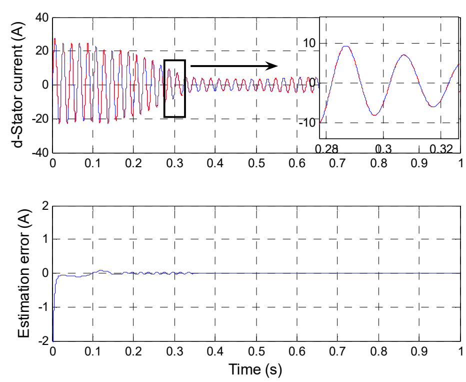

Figure 5. Measured (red line) and observed (blue line) *d*-stator current components and the corresponding estimation error

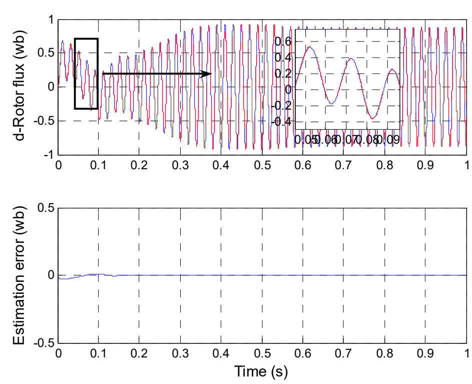

Figure 6. Measured (red line) and observed (blue line) *d*-rotor flux components and the corresponding estimation error

The simulation results of the designed and nonlinear H_∞ observer are presented in the following. Figure 5 and Figure 6 show the measured and estimated stator current and rotor flux components respectively. The Figure 7 and Figure 8 show the measured and estimated rotor angular velocity and the corresponding load torque respectively with corresponding estimation error. One can see that the estimated state variables of the machine follow the desired trajectories.

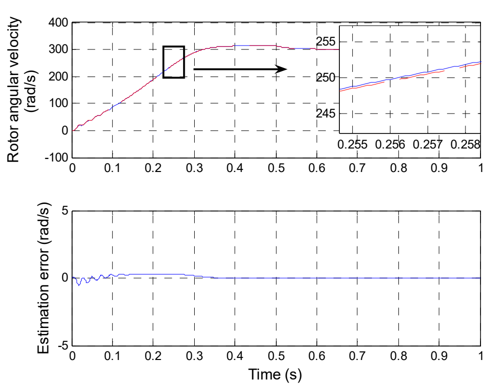

Figure 7. Measured (blue line) and observed(red line) rotor angular velocity of the (IM) and the corresponding estimation error

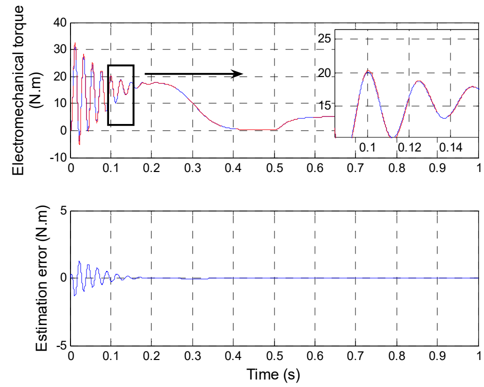

Figure 8. Measured (red line) and estimated (blue line) electromechanical torque of the (IM) and the corresponding estimation error

According to the Figure 7, we note a good trajectory tracking of the speed in the presence of disturbances and show the high performance of the observer H_∞. The simulation results demonstrate that, despite the partly unknown transition probabilities, the designed H_∞ filters are feasible and effective, ensuring the error systems are stochastically stable.

6. CONCLUSION

This paper gives a comparative study between the nonlinear observer based on circle criterion and nonlinear H_∞ observer.
The performance of the nonlinear observers is evaluated in terms of their ability to cope with model imperfections and process uncertainties such as measurement errors and uncertain initial conditions.

The advantages of the circle-criterion approach are the global Lipschitz restrictions removing and high gain avoiding. However it introduces linear matrix inequality (LMI) conditions.
From the simulation results, the proposed H_∞ observer has proved to be more robust than the observer based circle criterion when load variations of the IM occur, in the presence of disturbances.

On the other hand, the results obtained using the nonlinear H_∞ observer show that three characteristics can be obtained simultaneously. Asymptotic stability, robustness against nonlinear uncertainty and minimized guaranteed H_∞ cost.

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