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q-Deformed solitary pulses in the higher-order nonlinear Schrödinger equation with cubic-quintic nonlinear terms

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ABSTRACT

We investigate the existence and propagation properties of deformed solitary pulses in a non-Kerr medium described by the higher-order nonlinear Schrödinger equation with cubic-quintic nonlinear terms and third-order dispersion. Two different types of exact analytical q-deformed soliton solutions have been derived by means of the ansatz method. The results show that both width and amplitude of the soliton structures are influenced by the deformed factor. It is found that the introduced deformed factor lets the soliton solution deviates from the standard profile. The requirements on the parameters of the non-Kerr material for the existence of these localized structures are presented. By employing numerical simulations, we demonstrate the stability of these deformed soliton solutions under the finite perturbations. Finally, the collision between similar soliton pulses is also investigated.

1. Introduction

The propagation dynamics of an intense light pulse in a nonlinear fiber medium leads to many important phenomena displaying both fundamental and applied interests. Among the several nonlinear phenomena, the propagation of soliton pulses becomes a subject of significant interest in nonlinear science and particularly in nonlinear optics. Previous studies on soliton dynamics mostly concentrated on Kerr medium which is the manifestation of a non-resonant interaction and a trivial non-linear coefficient [1–5]. In this situation, the cubic nonlinear Schrödinger equation (NLS) has been successfully used to describe the short pulse propagation in Kerr media [6–10]. However, in a material possessing highly nonlinear susceptibilities, the higher order nonlinearities occur even at moderate pulse intensity [11–19]. One of the simplest cases are materials with quintic susceptibilities $\chi^{(5)}$, such as semiconductors or certain transparent organic materials [17,20] which are often modelled by the NLS family of equations incorporating additional higher-order terms to the cubic model. A more accurate description of soliton pulse propagation in such highly nonlinear optical material is to consider cubic-quintic nonlinearities instead of the usual Kerr nonlinearity [13–16,21–24]. As a widely studied model, Radhakrishnan, Kundu, and Lakshmanan (RKL) proposed a class of cubic-quintic nonlinear Schrödinger (CQNLS) equation to describe ultrashort light pulse propagation in non-Kerr media [25]. We should note that some important results have been obtained with previous investigations focused on finding some analytical solutions for CQNLS equation in different forms [13,14,23–29]. It is worthy to mention that the first investigation of soliton solutions in the CQNLS equation, particularly in the RKL model, has been realized by Hong et al. [26]. In such study, the authors have found the analytical kink-type solitary wave solutions of the RKL model by means of

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the coupled amplitude-phase formulation. Subsequently, Hong [27] has thoroughly used the ansatz method to obtain the combined bright-dark soliton solutions for the RKL model under certain parametric conditions. In Ref. [29], the Jacobi elliptic function method has been employed to derive the exact analytical solitary wave solutions for the RKL model. By suggesting the CQNLS equation extended to septic nonlinearity, an analytic expression for the MI gain have been obtained and shown to be sensitive of the septic nonlinearity in Ref. [30]. Recently, Liu et al. [31] used the ansatz method to find exact bright and dark quasi-soliton solutions governed by CQNLS equation with variable coefficients and investigated their interaction. Also, Triki et al. [32] presented three new types of nonlinearly chirped W-shaped soliton solutions which may exist in negative index materials exhibiting higher-order effects such as pseudo-quintic nonlinearity and self-steepening effect. However, for all investigations mentioned above, they are concerned with analytical soliton solutions expressed in terms of the standard secant hyperbolic function.

In recent years, much attention has been directed towards the study of deformed soliton solutions whose amplitudes may deviate from the standard form. Particularly, a set of exact nonautonomous deformed-soliton solutions and exact centre-of-mass positions of the solitons has been derived in Ref. [33], which describe some new soliton characteristics of an attractive Bose–Einstein condensate (BEC). In addition, Li et al. [34] have presented new nonautonomous deformed-soliton solutions for several different forms of the time-dependent atom–atom interaction and external parabolic potential governed by a one-dimensional nonautonomous Gross–Pitaevskii system (GP). Furthermore, Tao et al. [35] have constructed the deformed-soliton, breather, and rogue wave solutions of an inhomogeneous NLS equation by using the 1-fold Darboux transformation (DT). More recently, Cihan et al. [36] have derived the self-localized one- and two-soliton solutions of the NLS equation with a q-deformed Rosen–Morse potential by implementing a Petviashvili method (PM), and investigated the temporal behavior and stabilities of these new solitons. To the best of our knowledge, exact analytic deformed soliton solutions to the RKL model have been absent. It is of interest to study the existence of this special type of localized pulses in a non-Kerr medium governed by such higher-order NLS-typed equation. In this work, we derive exact analytic deformed soliton solutions to the RKL model by using the ansatz method. The obtained results show that a diversity of deformed localized solutions can be formed in the system, including dark, bright and W-shaped deformed solitary pulses.

The paper is organized as follows. In Sec. II, we present the RKL model describing the propagation of femtosecond light pulses in non-Kerr media, and we derive the evolution equation that governs the dynamics of pulse amplitude in the system. In Sec. III, we introduce two special amplitude ansatz which enables one to find families of novel deformed solitary wave solutions of the model. In Sec. IV, we analyse the dynamic behaviours and stability of the solutions by numerical simulation. Next, the Sec V is reserved to study the collision between similar pulses. Finally, we summarize our results in Sec. VI.

2. Model and amplitude equation

The RKL equation model with cubic-quintic nonlinear terms describing femtosecond light pulse propagation through non-Kerr media takes the form [25–29].

$$iA_z + A_{tt} + 2|A|^2A + i\alpha_1 A_{ttt} + i\alpha_2\left(|A|^2A\right)_t + i\alpha_3\left(|A|^4A\right)_t + \alpha_4|A|^4A = 0 \tag{1}$$

where $A(t,z)$ is a complex function representing a normalized complex amplitude of the pulse envelope, z is the normalised coordinate along the propagation direction of the carrier wave, and t is the retarded normalised time. The coefficients α_1 and α_2 are real parameters related to the third-order dispersion (TOD) and self-steepening, respectively. Also, the terms related to coefficients α_3 and α_4 stand for the quintic non-Kerr nonlinearities.

When the terms related to α_3 and α_4 of Eq. (1) are neglected, the resulting equation corresponds to the RKL model with Kerr law nonlinearity, which includes combined effect of shock and third-order of velocity dispersion. Quite recently, exact solitary wave solutions of the bell, kink and algebraic types for Eq. (1) have been obtained by means of the sub-ODE method in [37]. Moreover, many different exact soliton and periodic solutions for Eq. (1) have been derived using the first integral and direct algebraic methods in [38]. But here we are concerned with exact deformed-type soliton solutions of Eq. (1) in the presence of all dispersion and nonlinear terms.

To start with, we adopt the complex envelope traveling-wave solutions of the form [39].

$$A(z,t) = U(\xi)\exp(i\phi(z,t)) = U(t+\lambda z)\exp(i(kz-\omega t)) \tag{2}$$

where $U(\xi)$ is an unknown envelope function (assumed to be real), and λ is a real constant.

Substituting Eq. (2) Into Eq. (1) and separating the real and imaginary parts, we get

$$U_{\xi\xi} + \frac{(-k-\omega^2-\alpha_1\omega^3)}{(1+3\alpha_1\omega)}U + \frac{(2+\alpha_2\omega)}{(1+3\alpha_1\omega)}U^3 + \frac{(\alpha_4+\alpha_3\omega)}{(1+3\alpha_1\omega)}U^5 = 0 \tag{3}$$

$$U_{\xi\xi\xi} + \left[\left(\frac{\lambda-2\omega-3\alpha_1\omega^2}{\alpha_1}\right) + \frac{3\alpha_2}{\alpha_1}U^2 + \frac{5\alpha_3}{\alpha_1}U^4\right]U_\xi = 0) \tag{4}$$

If we derive Eq. (3) with respect to ξ, the resulting equation becomes

$$U_{\xi\xi\xi} + \left[\frac{(-k-\omega^2-\alpha_1\omega^3)}{(1+3\alpha_1\omega)} + \frac{3(2+\alpha_2\omega)}{(1+3\alpha_1\omega)}U^2 + \frac{5(\alpha_4+\alpha_3\omega)}{(1+3\alpha_1\omega)}U^4\right]U_\xi = 0 \tag{5}$$

Eqs. (4) and (5) are identical if we equate the coefficients of U^0, U^2 and U^4. By solving the resulting equations, one gets the following wave parameters:

$$\omega = \frac{2\alpha_1 - \alpha_2}{2\alpha_1\alpha_2} \tag{6}$$

$$k = 8\omega^2 + 8\alpha_1\omega^3 + \frac{1}{\alpha_1}((-3\lambda\alpha_1 + 2)\omega - \lambda) \tag{7}$$

together with

$$\frac{\alpha_3}{\alpha_2} = \frac{1}{2}\alpha_4 \tag{8}$$

which shows that the parameters α_2, α_3 and α_4 are not independent and the corresponding deformed soliton solutions are obtained in the framework of this relationship.

Further substitution of the relation (6) into (3) yields the following evolution equation

$$U_{\xi\xi} = a_1 U + a_2 U^3 + a_3 U^5 \tag{9}$$

with

$$a_1 = -\left(\frac{1}{4\alpha_1{}^2} - \frac{3}{\alpha_2{}^2}\right) - \left(\frac{\lambda}{\alpha_1} + \frac{1}{\alpha_1\alpha_2}\right), \quad a_2 = -\frac{\alpha_2}{\alpha_1}, \quad a_3 = -\frac{\alpha_4\alpha_2}{2\alpha_1}, \tag{10}$$

Eq. (9) describes the dynamics of the field amplitude in the non-Kerr medium. Multiplying Eq. (9) by U_ξ and integrating once with respect to the variable ξ, we obtain

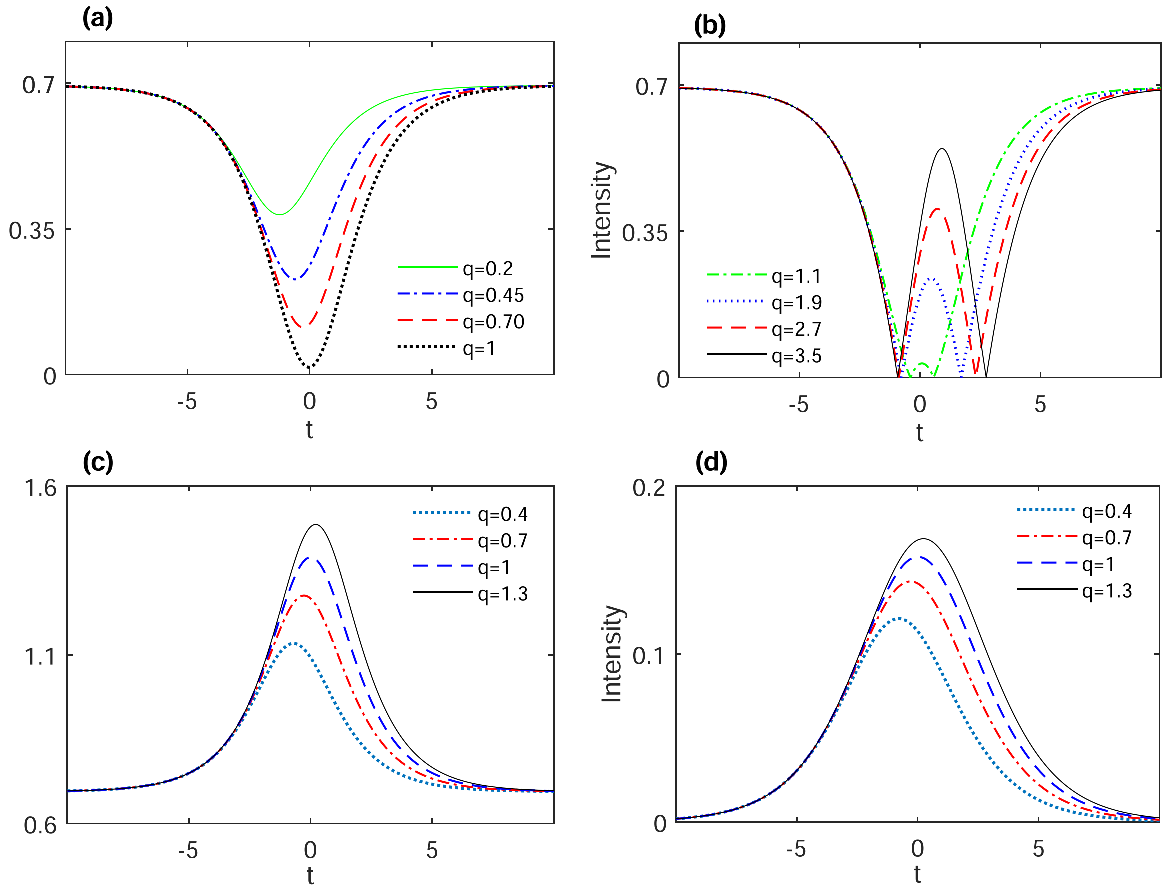

Fig. 1. **:** The q-deformed soliton solutions (23) and (31) of RKL model (1) for several values of deformation factor q. (**a**); (**b**): The soliton solutions (23) with $\mathscr{E} = -\sqrt{q}$ and $q \le 1$; $q > 1$, respectively. (**c**): the soliton solutions (23) with $\mathscr{E} = +\sqrt{q}$ and $q > 0$. (**d**): the soliton solutions (31) with $q > 0$..

$$(U_\xi)^2 = a_1 U^2 + \frac{a_2}{2}U^4 + \frac{a_3}{3}U^6 \quad + 2E \tag{11}$$

where E is an integration constant to be determined which coincides with the energy of anharmonic oscillator [39].

Eq. (11) is known to admit a diversity of closed form solutions such as kink, solitary and periodic wave solutions in the case when the integration constant E has a zero $(E = 0)$ [40] and nonzero values$(E \neq 0)$ [41]. In the following, we report the first analytical demonstration of existence of q-deformed soliton solutions on a continuous-wave (cw) background for Eq. (11) in the general case when $E \neq 0$.

3. *q*-Deformed solitary wave solutions

First, we start our analysis by setting the amplitude function as $U = u^{1/2}$. This change of variable transforms Eq. (11) into an elliptic equation having a fourth-degree nonlinear term as

$$\frac{1}{4}(u_\xi)^2 = a_1 u^2 + \frac{a_2}{2}u^3 + \frac{a_3}{3}u^4 \quad + 2Eu \tag{12}$$

Localized pulse solutions are achieved by solving Eq. (12) for different values of E. Below, we will show that this equation admits a novel class of deformed solitonic solutions in the most general case, when the energy E and the coefficients $a_i (i = 1 - 3)$ have nonzero values. To this end, we will introduce a specific ansatz with a deformation factor, whose asymptotic value is nonzero when the traveling wave variable tends to infinity.

3.1. W-shaped, dark and bright q-Deformed soliton on a cw background

To obtain exact deformed soliton solutions for the RKL model (1), we introduce a special ansatz of the form

$$u(\xi) = \beta + \rho \operatorname{sech}_q(\eta\xi) = \quad \beta + \frac{2\rho}{e^{\eta\xi} + qe^{-\eta\xi}} \tag{13}$$

where β, ρ and η are unknown parameters to be determined, while q represents the deformed factor. Here the parameter β decides the strength of the background, in which this structure propagates in the non-Kerr medium.

Expression (13) indicates that the amplitude of solitary wave solutions obtained based on this ansatz do not approach zero when the traveling coordinate approaches infinity ($|\xi| \to \infty$). Here, the deformation factor $q \neq 0$ allows the soliton solution to deviate from the standard shape, so that it is not confined to a single form, even the amplitude settings are not changed, as shown in Fig. 1(a-c). Thus, we can obtain different shapes of deformed soliton solutions depending on the value of the factor q, which influences the dynamical behavior of the obtained deformed localized waves.

Now, substituting Eq. (13) into Eq. (12) and then setting the coefficients of $\operatorname{sech}^m{}_q(\eta\xi)$ (where $m = 0, \ldots 4$) to zero, we get the following set of algebraic equations:

$$\rho^2\left[\frac{q}{4}\eta^2 + \frac{a_3}{3}\rho^2\right] = 0 \tag{14}$$

$$\beta\left[a_1\beta + \frac{a_2}{2}\beta^2 + \frac{a_3}{3}\beta^3 + 2E\right] = 0 \tag{15}$$

$$\rho\left[2a_1\beta + \frac{3a_2}{2}\beta^2 + \frac{4a_3}{3}\beta^3 + 2E\right] = 0 \tag{16}$$

$$\rho^2\left[a_1 + \frac{3a_2}{2}\beta + 2a_3\beta^2 - \frac{1}{4}\eta^2\right] = 0 \tag{17}$$

$$\rho^3\left[\frac{a_2}{2} + \frac{4a_3}{3}\beta\right] = 0 \tag{18}$$

Obviously, Eqs. (16), (17) and (18) can be solved self-consistently for the unknown parameter β, ρ and η to give

$$\eta^2 = -\frac{3{a_2}^2}{16a_3} \tag{19}$$

$$\beta = -\frac{3a_2}{8a_3} \tag{20}$$

$$\rho = \pm\beta\sqrt{q} \tag{21}$$

Along with the parametric conditions

$$a_1 = \frac{15{a_2}^2}{64a_3}, \quad 2E = \frac{9{a_2}^3}{256{a_3}^2} \tag{22}$$

If we insert the solution (13) into Eq. (2), we obtain an exact deformed solitary pulse solution for the RKL model (1) as

$$U(z,t) = \sqrt{\beta\left[1 + \mathscr{E}\mathrm{sech}_q\left(\sqrt{-\frac{3{a_2}^2}{16a_3}}\xi\right)\right]}e^{i(kz-\omega t)} \tag{23}$$

With

$$\mathscr{E} = \pm\sqrt{q}.$$

From the relation (19), one must require $a_3 < 0$ for the pulse width η to be real. In addition, we should have $a_1 < 0$ and $a_2 > 0$ as follows from Eq. (22). This implies that the deformed soliton solution on a cw background (23) exists provides that the material parameters satisfy the conditions $\alpha_1\alpha_2 < 0$ and $\alpha_4 < 0$. Physically, Eq. (23) describes the propagation of three different types of deformed solitary pulse solutions for the RKL model (1). When the deformation factor $q \le 1$, the solution (23) represents a deformed dark pulse if the lower sign is considered ($i.e$, $\mathscr{E} = -\sqrt{q}$). Moreover, the solution takes the form of a bright pulse on a cw background when we consider the upper sign in (23) expression ($i.e$, $\mathscr{E} = +\sqrt{q}$). In the case of $q > 1$, the solution (23) with the lower sign becomes a *W*-shaped deformed soliton. Thus, one can conclude that the pulse shape obtained based on the ansatz (13) is completely determined by the deformation factor q and can takes the bright, dark and *W*-shaped deformed solitary waves, as clearly seen in Fig. 1(a-c).

3.2. Bright q-Deformed soliton on a zero background

Now let us adopt another ansatz solution which describe a deformed-soliton on a zero background as

$$u(\xi) = \frac{A}{B + cosh_q(\eta\xi)} = \frac{2A}{2B + (e^{\eta\xi} + qe^{-\eta\xi})} \tag{24}$$

In the case of a zero energy ($E = 0$). Here A and η are unknown parameters related to the amplitude and width of the solitary wave. Substituting the ansatz (24) into Eq. (12) and setting the coefficients of the ${\cosh^m}_q(\eta\xi)$ equal to zero, we obtain

$$A^2\left[\frac{\eta^2}{4} - a_1\right] = 0 \tag{25}$$

$$A^2\left[B\eta^2 + a_2A\right] = 0 \tag{26}$$

$$A^2\left[\frac{\eta^2}{4}(B^2 - q) - \frac{a_3A^2}{3}\right] = 0 \tag{27}$$

Obviously, Eqs. (25), (26) and (27) yields the following solitary wave parameters:

$$\eta^2 = 4a_1 \tag{28}$$

$$A = -\frac{4a_1}{a_2}B \tag{29}$$

with

$$B = \pm\sqrt{q}\left[1 - \frac{16a_1a_3}{3{a_2}^2}\right]^{-1/2} \tag{30}$$

Making use of the above results, we find that the exact deformed solitary wave solution of Eq. (1) of the form

$$U(z,t) = \left[\frac{A}{B + cosh_q\left(2\sqrt{a_1}\,\xi\right)}\right]^{1/2}e^{i(kz-\omega t)} \tag{31}$$

provided that $a_1 > 0$ and $a_3 < \left|\frac{3{a_2}^2}{16a_1}\right|$.

From Eqs. (28), (29) and (30), we clearly see that the pulse width η does not depend upon the deformation factor q, while the q dependence arises through the parameters A and B. Physically, Eq. (31) describes a bright deformed solitary wave on zero background for RKL model (1), which differs from that reported in Refs. [46–48] by exhibiting a shift shape from its original profile due to the q factor.

It is interesting to compare the bright deformed solitary wave solution (31) with the solitary waves (12) and (13) obtained for the higher-order NLSE models governing ultrashort pulse propagation in dual-power law media [44] and cubic-quintic-septic media with weak nonlocality [45], respectively. One can see that when the deformation factor takes the value $q = 1$, the intensity of the deformed solitary wave solution (31) takes the same functional form as those obtained in Refs. [44,45]. The presence of the deformation factor q in the solitary wave solution (31) allows the nonlinear wave to deviate from the standard shape and leads to a variety of pulse profiles. A noteworthy characteristic is that the present solitary wave (31) has a free parameter λ similarly to those reported in Refs. [44,45]. We should note here that the parameters of this solitary waves such as amplitude, inverse temporal width, and wave number are all determined with both the free parameter λ and the system parameters such as the nonlinear coefficients α_2, α_3 and α_4 and the dispersion parameter α_1. It should be noticed that the formation of this deformed solitary wave is due to the interplay between second- and third-order dispersions, cubic nonlinearity, quintic non-Kerr nonlinearities, and self-steepening effect, which have a significant influence on its characteristics and propagation dynamics.

4. Dynamic behavior and stability analysis

In the above section, we obtained two types of q-deformed solitary waves that can be formed in a non-Kerr nonlinear medium governed by the the RKL model (1), which are the structures (23) and (31). Now, we will discuss the dynamic behavior of these deformed localized waves for different values of the deformation factor q.

Fig. 2(a-c) display the dynamic behaviors of the deformed solitary waves given by Eq. (23) for $\mathscr{E} = \pm\sqrt{q}$. Firstly, we take $\mathscr{E} = -\sqrt{q}$, for both cases of $q \leq 1$ and $q > 1$. In the first case a deformed dark soliton occurs [Fig. 2(a)], whereas a W-shaped deformed soliton arises in the second case [Fig. 2(b)]. However, as the q-factor approaches ~ 1, the solitonic width broadness continuously so that the depth of the solitonic background simultaneously growths, as shown in Fig. 1(a). Hence, if the factor q continues to deviate absolutely from the value 1, the solitonic width expands continually, while its depth quickly converts to peaks, in which the W-shape soliton pulses begin to arise, as plotted in Fig. 1(b). Likewise, we observe that, this last type of soliton pulses is all the more stable despite the impact of the deformation factor q on its evolution, as displayed in Fig. 2(b). Unlike the case (i), if we take $\mathscr{E} = +\sqrt{q}$, the solution (23) represents a q-deformed bright soliton on cw background, as shown in Fig. 2(c). We note that, for different values of the factor q, only the peak and the platform of the solitons increase when the factor q varies more than zero, which we have presented in Fig. 1(c). It is interesting to note that, the profile of the pulses remains unchanged during the evolution, except for some time-shifts which appear due to the group velocity adjustment related to the parameter λ, as clearly shown in the Fig. 2(c) with $\lambda = -5$.

Let us now consider the second type of deformed solitary wave obtained above [Eq. (33)], which is depicted in Fig. 2(d). From this figure, we see that the solution (31) represents a bright deformed soliton on zero background. Obviously, the strain factor causes the solutions to deviate longitudinally from their standard shape, while their amplitude increases, as shown in Fig. 1(d). Unlike the previous cases, it can be seen that during the propagation distance, receded waves occur due to the positive sign of the constant λ what produces an immutable time shift in propagation, while the solitary velocity remains unchanged, as shown in Fig. 2(d).

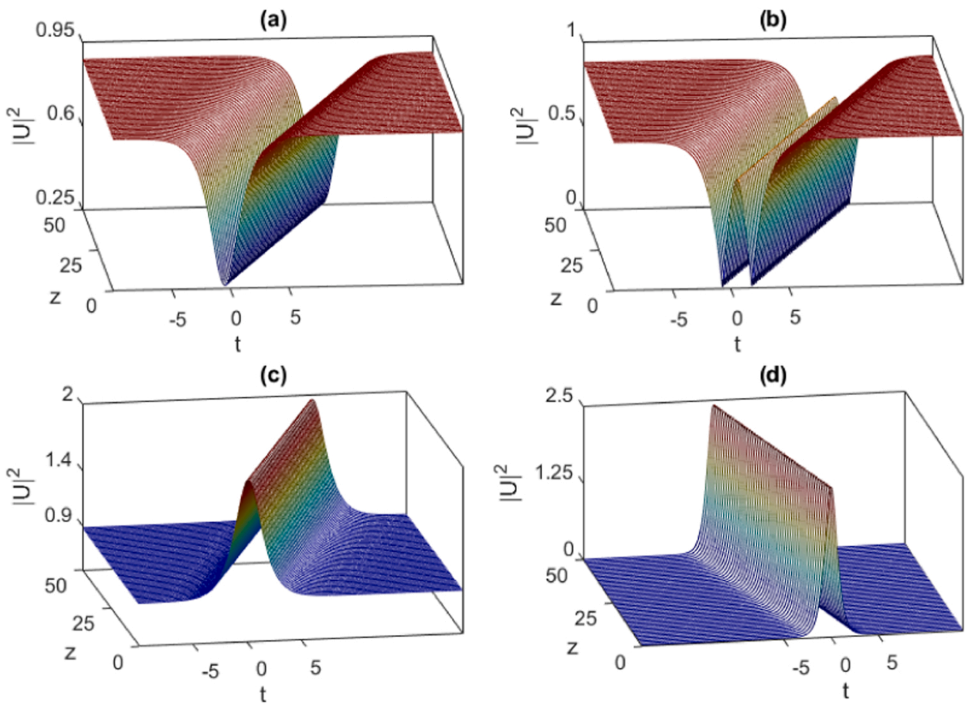

Fig. 2. Evolution plot of the q-deformed solitons as computed from Eq. (23) for the values: $\alpha_1 = -0.3, \alpha_2 = 1.5, \alpha_3 = -0.6, \alpha_4 = -0.8, \lambda = -5$. (**a**): $\mathscr{E} = -\sqrt{q}$ & $q = 0.5$. (**b**): $\mathscr{E} = -\sqrt{q}$ & $q = 3$. (**c**): $\mathscr{E} = +\sqrt{q}$ & $q = 1.5$. (**d**): Evolution plot of the q-deformed soliton as computed from Eq. (31) for the values: $\alpha_1 = -0.5, \alpha_2 = -1, \alpha_3 = -0.4, \alpha_4 = 0.8, \lambda = 2$ with $q = 1.5$..

In the following, we will study the robustness of the obtained structures to verify their ability to propagate in a disturbed environment over an appreciable distance. To demonstrate the stability of the deformed solitary wave structures presented above, we numerically simulate Eq. (1) under the impact of white noise. To this end, we produced a photonic noise, which corresponds to 0.02% of the average power of the input profile. Then, we used the split-step Fourier method (SSFM) using the same parameter's values of Fig. 2. The results of the simulation are presented in Fig. 3. We can clearly see that, for the two types of q-deformed solitary waves presented above, the initial profile quickly goes to the profile of the exact solution (23) and (31), despite some small periodic oscillations attached to the soliton. So, it can be avowed that no principal difference to Fig. 2 occurs. We can conclude that, even the system is strongly disturbed by the white noise in existence of the deformation factor, this provides solitons displaying very high stability.

5. Soliton collisions

Practically, there are many effects which can contribute to the instability of soliton robustness. It is important to involve the soliton in a more powerful test than the noise perturbation, such as the collision with equivalent pulses. We note that the collision of adjacent solitons presents the most beautiful process of soliton phenomena. Based on their relative phases, there are two categories: coherent and incoherent collisions [42]. Noting that, the coherent collisions exist when the pulses overlap in the presence of a nonlinear medium response to interference effects. Unlike, the incoherent collisions occur when the time varies much slower than the relative phase between pulses [43]. In general, soliton collisions process is very complicated, which necessitates the use of numerical simulations [49, 50].

To overcome mutual interference in the soliton transmissions, we need to use the elastic interaction between optical solitons. Often, for an optical fiber in the real environment, the transmission of the solitons is described by the nonlinear Schrodinger equation (NLS) with variable coefficients which lets us to study the interaction between the adjacent solitons under the influence of fiber parameters [51]. For studying the collision dynamics of deformed solitons pulses governed by the RKL equation model, we use the following superposition of two soliton solutions profiles (23) with equal amplitudes

$$U(0,t) = U(0,t-q_0) + U(0,t+q_0)\exp(i\theta) \tag{32}$$

Where θ represents an initial relative phase among the two temporally soliton pulses initially separated by a distance q_0..

Firstly, we study the case when the relative phase has a zero value ($\theta = 0$), which corresponds to the simple collision between two similar solitons initially launched in parallel. We perform the collision among soliton pulses (31) and (23) using the same parameter values as in Fig.2 and a separation distance $q_0 = 3$. The results are presented in Fig. 4. From this figure, we see that when the soliton pulses collide coherently, the intensity in the central collision zone growths, which leads to an attraction of more light towards the

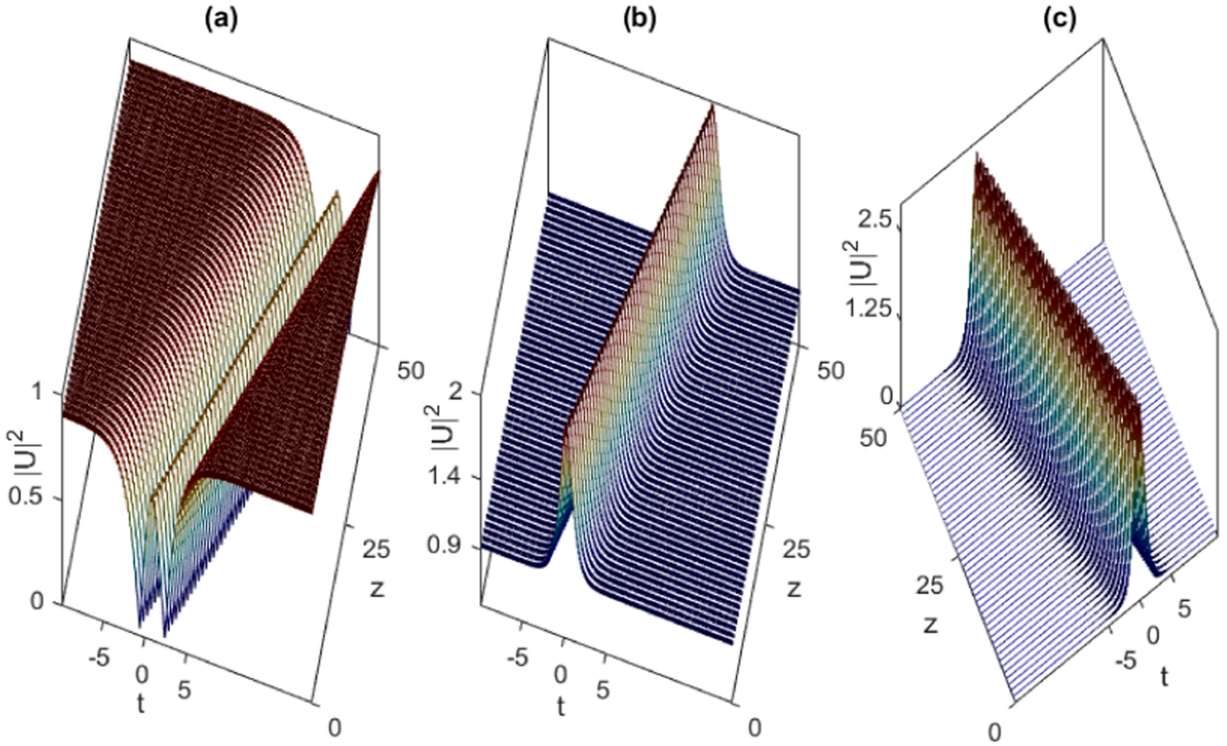

Fig. 3. The exact solutions under the perturbation of white noise with the same parameters as in Fig. 2. (**a**): The q-deformed W-shape soliton. (**b**): The q-deformed bright soliton on cw- background. (**c**): The q-deformed soliton solution of Fig. 2 (**d**) evolved with added noise.

centre. Therefore, the soliton evolution shows that the force is certainly attractive but there is no exchange during the collision, as shown in Fig. 4(a), 4(b) and 4 (c). The same behavior is observed when we perform the collision process for the second case of soliton solutions (31), where only the intensity of the resulting collided pulses increases when the amplitude function changes, relative to that of aexpression (23) [see Fig. 4(d)]. Also, these localized structures keep their velocity and shape throughout the propagation media. The scenario is different if we choose the relative phase $\theta = \pi$, where the solitonic pulses approach each other and just at the central region of collision they interfere destructively. Afterward, the two solitons regenerate and the centroid of each pulse moves external in which the solitons seem to repel, as clearly observed in Fig.5(a), with the case of solution (31). Under this condition, the pulses present an unstable character differently to the previous situations.

By using the Hirota's bilinear method, Zhou et al. in Ref. [52], presented the optical soliton amplification effect in high power transmission systems for the variable-coefficients NLS equation. Based on three soliton interactions in a nonlinear fiber with decreasing dispersion and periodically lumped amplification, the influence of parallel and nonparallel propagation of optical solitons on optical soliton amplification is carefully addressed. Besides to this study, we have a more complicated situation when the three-pulses collision occurs with other relative phases based on a numerical method. In Fig. 5(b) and Fig.5(c) we exhibit the scattering behaviour of three deformed solitary pulses (23) with $\mathscr{E} = +\sqrt{q}$, in both relative phases of middle pulse $\theta = \pi$ and $\theta = \pi/2$, repectively. It can be seen that, the collision between the second (middle) pulse and the last (right side) also occurs destructively. Then, the last pulse survives and produces a non-trivial exchange energy with the first (left side) pulse which eventually results in a repulsive force that causes the pulses to diverge, as shown in Fig.5(b). By taking the relative phase $\pi/2$, an interesting collision dynamic is identified, where the last soliton pulse (right side) crosses the two trajectories of other pulses which propagate in parallel. This results in an exchange of energy during the collision process, but with different peaks of intensity, as shown in Fig.5(c). In summary, the collision process can provide a sufficient complementary study on the robustness of this type of solitons or their ability to perform very stable propagation in highly perturbed environments.

6. Conclusion

To summarize, we have presented two types of q-deformed soliton solutions for the higher-order nonlinear Schrödinger equation with cubic–quintic nonlinear terms, describing femtosecond pulse propagation in non-Kerr media. The solutions include dark, W-shaped and bright deformed soliton solutions on both zero and nonzero background. These classes of localized pulses have been obtained by means of two special ansatzes involving a deformation factor, which allows the resulting soliton solution to deviate from the standard shape and leads to a variety of pulse profiles. It is found that the contribution of all dispersive and nonlinear effects is an important feature to form these structures. The conditions on the parameters of non-Kerr media for the existence of these solitary waves are also reported. These constraints show a subtle balance among second and third-order dispersions, self-steepening, and cubic-quintic nonlinear terms, which have a deep implication in controlling the soliton pulse dynamics. Also, we demonstrated stable

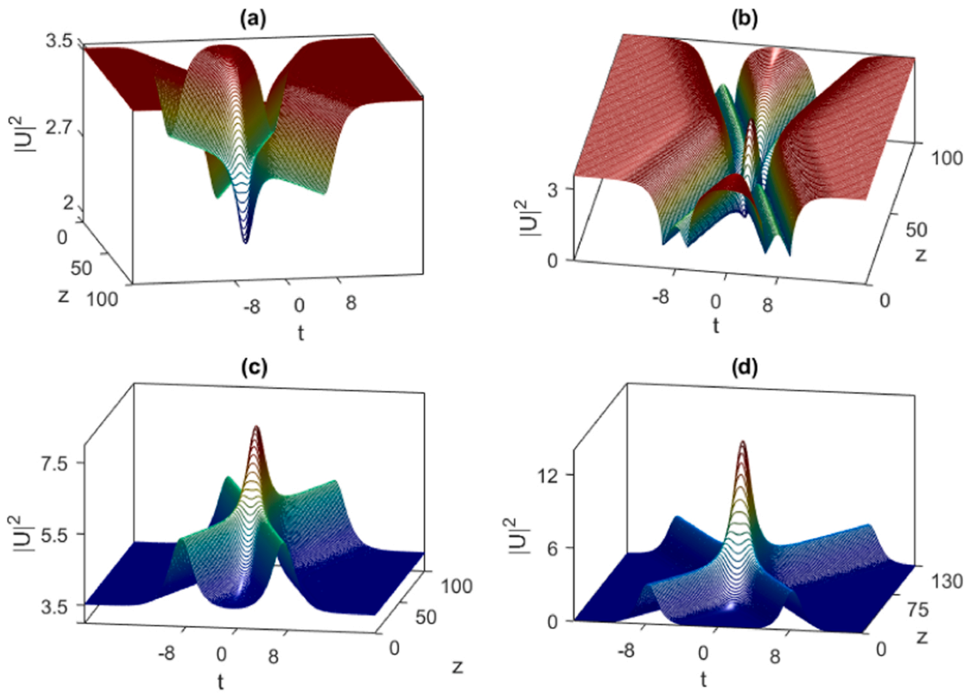

Fig. 4. : The dynamical evolution of two colliding solitons pulses (23) and (31) when the initial separation $q_0 = 3$ and a relative phase $\theta = 0$. The adopted parameters are the same as in Fig. 2.

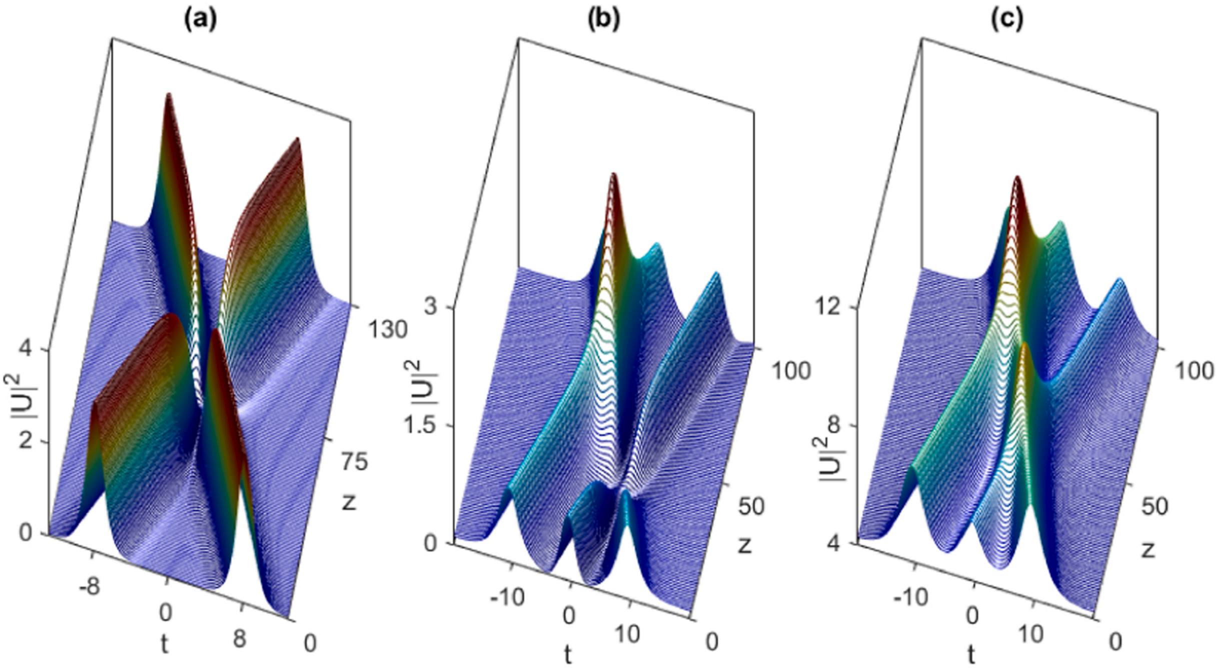

Fig. 5. **:** The dynamical evolution of two and three colliding soliton pulses (23) and (31) when the initial separation is $q_0 = 3$ and a relative phase $\theta \neq 0$. (**a**): Two similar solitons (31) with $\theta = \pi$. (**b**);(**c**): Three similar solitons (13) with $\theta = \pi$ and $\theta = \pi/2$, repectively. The parameters for (**a**) are the same as in Fig. 2(d). For (**b**) and (**c**), as in Fig. 2(**c**).

propagation over long distance even in the presence of external perturbation, which is seeded in the form of both white noise and coherent collision between similar pulses. Very robust solitons are then obtained, exhibiting great stability under the power of a strong noise, and capable to subsist after a collision process. We hope that the results presented here may help in stimulating more research in understanding the deformed localized wave in systems with cubic-quintic nonlinearities and may be useful to study the evolution dynamics of nonlinear waves in fibers, waveguides and Bose–Einstein condensates.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

The data that has been used is confidential.

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