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Robust controllers design for constrained nonlinear parameter varying descriptor systems

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Abstract

This article proposes a method for designing robust controller laws for a class of uncertain nonlinear parameter varying (NLPV) descriptor systems under input saturation and external disturbances. Both static and dynamic output feedback controllers are proposed. To synthesize the fuzzy controllers, the stability conditions are derived using polytopic parameter-dependent (PD) nonquadratic Lyapunov functions with respect to the given saturation constraint on the control input. First, the designed conditions are established in terms of linear matrix inequalities (LMIs) and $\mathfrak{L}_2$ gain performance is used to attenuate the effect of the external disturbance signals. Then, the estimation of the largest domain of attraction (DoA) for the system is formulated and solved as an optimization problem. Two examples are used to illustrate the effectiveness of the proposed design methods.

KEYWORDS

dynamic output feedback controller, input saturation, linear matrix inequalities, nonlinear parameter varying descriptor models, robust controllers, static output feedback controller, uncertainties

1 | INTRODUCTION

In industrial applications, the nonlinear parameter varying (NLPV) systems require obtaining better control performances such as automotive transmission systems, steeper motor drives, computer disk drives, some robot control systems, network control robotic manipulations, and near-space vehicle control systems.[1–3] The control performances involve different operating conditions due to the continuous increase of the productivity of modern industrial systems. However, controlling strategies has become more difficult in recent years, especially when the unknown inputs/uncertainties occur, whatever the origin of the unknown inputs, that is, external and internal disturbances, saturation of actuator/sensor or installation, faults, and so on. Furthermore, the system behavior can change in most cases, and this undesirable behavior may degrade the performances and lead to system instability. Recently, the control design of dynamic nonlinear systems, particularly the classes of NLPV and Takagi–Sugeno (T-S) models, has attracted a lot of attention. Indeed, output feedback stabilization,[4,5] input–output-to-state stability problem,[6,7] and sliding mode control[8–10] have been studied for NLPV and T-S models. In order to guarantee the stability and improve the performance of NLPV descriptor systems, the techniques of designing control laws have been developed by the scientific community, and have received growing interest. Moreover, stability analysis and stabilization of NLPV models for the design of observers and control laws are addressed in many works.[11–13] The stabilization technique based on the LMI conditions principle is one of the most commonly used approaches in control design field.

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Control design for nonlinear T-S systems has been studied using a quadratic Lyapunov functions, where the employed approaches present different degrees of conservatism since a common Lyapunov matrix should be found for a set of linear matrix inequalities (LMIs). To overcome this problem, various alternative classes of Lyapunov function candidates have been suggested such as piecewise Lyapunov function,[14] multiple Lyapunov function,[15] and parameter-dependent (PD) Lyapunov Function (see References 16 and 17 for a recent review). Moreover, the effectiveness of PD Lyapunov functions has been demonstrated for stability analysis, especially for discrete-time T-S systems. Furthermore, a wide range of T-S fuzzy methods based on observers can be found in the literature, for instance, by considering a symmetrical barrier Lyapunov function,[18] Lyapunov function based sliding mode,[19] and Lyapunov function with young's inequality.[20]

Physical constraints such as control input saturation and state constraints are ubiquitous in real-world applications due to safety and/or economic reasons. Saturation is a very mutual nonlinear phenomenon encountered in practice. It is considered as one of the most important nonlinearities that exist, practically, everywhere and in many practical control systems since the system actuators and sensors cannot provide signals of unlimited amplitude under the constraints of physics, security, and technology. Usually, it degrades the performance of the closed-loop system and leads to large overshoot. Consequently, the design of stabilization controllers for a dynamic system under input saturation has attracted the attention of several researchers.[21–23] So far, there are two main approaches to deal with the input saturation problem, the first one is a two-step approach consists of designing a nominal controller with ignoring actuator saturation, meeting the performance specifications, whatever the saturation effect. Then, an additional anti-windup compensator is introduced in order to minimize the performances of the closed-loop system, as a function of the difference between the unsaturated and saturated system control signals.[24–27] The second approach considers the saturation from the beginning of the controller design, and then the controller gains are adjusted accordingly to the saturation levels.[28–30] Among these strategies, one can cite the invariant sets framework, which has been significantly developed in control engineering over the last decades[31]; the convexity approach is based on the saturation function that is also dealing with saturated input constraints.[12,32] The main idea is to consider a bounded ellipsoidal symmetric region of stability by solving a set of LMIs constraints, and ensures that every state trajectory initialized inside a so-called invariant set remains inside this set. An interesting approach of the invariant set framework, applied in Reference 22, is to determine the largest invariant set by maximizing an estimate of the DoA of the closed-loop system.[31]

More recently, it has been shown that employing a descriptor approach to deal with discrete-time input saturated NLPV systems leads to less computational cost for the LMIs solution as well as less conservatism conditions.[30,33] For the control of real industrial systems, state feedback controls[22,23] assume the availability of all system state variables, which is not always realistic because it may be very expensive or even impossible to measure all process state variables, which, in most cases, are not directly adapted to output feedback control designs.[15] In this case, only partial information from measurable outputs can be considered. Several output feedback control approaches for classical/descriptor systems[5,34–38] have been developed: a first one consists of introducing an observer, making it possible to obtain a complete estimate of the state vector, based on the input and output signal measurements. An estimated state feedback control law or observer-based controller can be used as to allow the stabilization of a dynamic system.[11] Although this method is interesting in practice, it leads to an increase in the cost of online calculation due to the presence of the observer. Other types of approaches consist of synthesizing controllers using only the information available from the output vector. In this framework, two types of control laws can be considered: static output feedback controller (SOFC) and dynamic output feedback controller (DOFC).

In a large class of real-world applications, SOFC and DOFC strategies have been used by many authors in different control applications,[35,36,39–43] especially for time varying parameter dependency system,[41] and linear parameter varying system.[44] Their instability is caused by internal and external disturbances which is a fundamental problem in control engineering. In this article, the design of a new nonlinear SOFC and DOFC in the context of NLPV systems is envisaged. Some recent studies[32,34–36,40–41,45–46] deal with these controllers, and for a descriptor formulation, see, for example, References 37, 38 that have obtained research results through the use of these control laws. SOFC represents the simplest and easiest control structure because it does not require any online differential equation solution and, thus, reduces the computational cost. It is considered to be one of the easiest approaches to implement in practice since its earnings are calculated online for practical applications.[30,44] Unlike observer based command, DOFC is one of more convenient and flexible controller used in industrial processes. In fact, the synthesis of this type of controller for robust control framework ensures a reduced cost of online computation. Interesting results can be found in References 35, 41, 47.

Besides, external disturbances are often a source of instability and poor performance of nonlinear systems. In particular, the $\mathfrak{L}_\infty$ command for fuzzy systems is treated in References 13, 39, 48. The $\mathfrak{L}_2/\mathfrak{L}_\infty$ approach is used to analyze and synthesize controllers/observers obtaining an optimal level of disturbance attenuation (see, for example, References 6–8, and references therein). The results presented are illustrated through several examples.

Motivated by the above discussion, in this article, the output feedback controllers for uncertain NLPV descriptor systems subject to input saturation and external disturbances are designed. The saturation function is treated through a dead-zone nonlinearity satisfying a modified sector condition, which ensures that the resulting closed-loop systems are asymptotically stable with an estimation of the DoA described by the level set of the PD nonquadratic Lyapunov function, and the local stability is handled considering the largest ellipsoidal DoA inside a given polytopic region in the state space. A novel set of sufficient LMI conditions are developed aiming to design less conservative output feedback controllers. Two examples are presented showing favorable comparisons with recently published similar control design methodologies. The main contributions of this article are summarized as follows:

- For the control design, both SOFC and DOFC configurations are designed for uncertain discrete-time NLPV descriptor system, subject to input saturation and external disturbances. The controls design approach has a novel structure that depends on parameter-varying nonlinearity. With the use of PD nonquadratic Lyapunov function, the proposed controller will provide less conservative results such as larger DoA and better disturbance rejection capabilities. Compared to the approach presented in References 28, 38, 44, 47, the proposed controllers design method has the ability to preserve stability for open-loop unstable systems. Accordingly, a large class of nonlinear constrained systems can be considered.
- A PD of nonquadratic Lyapunov function is used for stability analysis.
- The $\mathfrak{L}_2$-gain is used to attenuate the effect of external disturbances and derived conditions of asymptotic stability in the presence of actuators saturation are established and solved by means of LMIs convex optimization.
- The problem of maximizing the estimate of the DoA is expressed and calculated as an optimization problem.

This article is organized as follows: Section 2 formulates the control problem statement, description of discrete-time uncertain NLPV descriptor system, and some useful preliminaries are also presented. SOFC and DOFC design conditions are presented in Sections 3 and 4, respectively. The effectiveness of the proposed methods is clearly demonstrated by means of examples in Section 5, and finally, Section 6 provides some conclusions.

2 | NOTATION AND PRELIMINARIES

2.1 | Notations

Throughout this article, the following notations are adopted to represent conveniently the different expressions, given a set of nonlinear function:$h_i(.)$, $v_{\hbar}(.)\,, i \in \{1 \ldots .r\}\,, \hbar \in \{1 \ldots .r_e\}$, are the nonlinear scalar functions. This work only focuses on measurable premise variables grouped in the vector $z(k)$, whose measurements can be obtained from the output $y(k)$, which is known and usually depend on the state vector, and can be equivalently represented by a vector of discrete-time varying parameter, expressed as $h_i(z(k))$ and $v_{\hbar}(z(k))$, satisfying the convex sum property.

These shorthand notation will be used in the sequel to represent convex sum property of discrete-time varying matrices expressions: $Y_h = Y_h(z(k)) = \sum_{i=1}^{r} h_i(z(k))\, Y_i$; $Y_v = Y_v(z(k)) = \sum_{\hbar=1}^{r_e} v_{\hbar}(z(k))\, Y_{\hbar}$, for single convex sum; $Y_{h-} = Y_{h-}(z(k)) = \left(\sum_{i=1}^{r} h_i(z(k))\, Y_i\right)^{-1}$ for the inverse of a convex sum; and $Y_{hh} = Y_{hh}(z(k)) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(k))\, h_j(z(k))\, Y_{ij}$ for a doubled rested convex sum. For a vector x and z, x_k symbolizes $x(k)$, z_k defined by $z(k)$ and x_{k+} by $x(k+1)$. I_r denotes the set $\{1, 2, \cdots, r\}$, and I_{r_e} denotes the set $\{1, 2, \cdots, r_e\}$, $\mathfrak{R}^+$ represents the set of positive real integer. $\mathcal{H}(A) = A + A^T$ is the hermitian of matrix A. $\mathbb{Z} + (*)$ denotes $\mathbb{Z} + \mathbb{Z}^T$. I denotes the identity matrix. An asterisk * symbolizes the symmetric block matrices.

The convex hull of these vectors is denoted as: $co\{x, y\} = \{\alpha_1 x + \alpha_2 y = 1, \alpha_1 + \alpha_2 = 1; \alpha_1 \geq 0; \alpha_2 \geq 0\}$.

2.2 Preliminaries

This section describes the considered class of uncertain NLPV descriptor system affected by actuator saturation and external disturbances. The NLPV descriptor system with bounded parameters can be represented as follows:

$$\begin{cases} \sum_{\hbar=1}^{r_e} v_{\hbar}(z_k)(E_{\hbar}+\delta E_{\hbar})x_{k+} = \sum_{i=1}^{r} h_i(z_k)((A_i+\Delta A_i)x_k + (B_i+\Delta B_i)sat(u_k)) + B_{\omega}\omega_k \\ y_k = Cx_k \end{cases} \tag{1}$$

where $x_k \in \Re^{n_x}, u_k \in \Re^{n_u}, y_k \in \Re^{n_y}, \omega_k \in \Re^{n_\omega}$ are the state, control input, output vector, and the exogenous disturbances, respectively. The PD state-space matrices: $E_{\hbar}(z_k)$, $A_i(z_k)$, $B_i(z_k)$, C, B_ω are of the appropriate dimensions, where $i \in I_r$ represent the ith linear right hand-side sub-model, $\hbar \in I_{r_e}$ is the $\hbar$th linear left-hand side sub-model of NLPV descriptor model (1). The matrix C involved in Equation (1) is assumed to be a full row rank, k is a current samples. It is assumed that all parameters z_k, where nl is a number of nonlinearities in the left/right hand side of system (1), depend on varying parameters and/or state variables, which are bounded, measurable, and valued in the domain of an hypercube such that[44]: $z_k \in \mathfrak{Q}$ and $z_{imin} \leq z_{imax}$ are known lower and upper bounds of z_k.We assume that E_v, A_h, and B_h of system (1) are continuous in the hypercube $\mathfrak{Q}$ and are in affine parameter dependence, are represented such that, $\forall z_k \in \mathfrak{Q}$:

$$\mathcal{Z}(z_k) = \mathcal{Z}_0 + \sum_{i=1}^{nl} z_i(k)\mathcal{Z}_i; \text{Where } \mathcal{Z}(z_k) = [\,E_v, A_h, B_h] \tag{2}$$

$$z(k) \in \Re^{nl} = \{z_i |\, z_{imin} \leq z_i(k) \leq z_{imax}, \forall k > 0\} \tag{3}$$

The NLPV descriptor system (1) with bounded parameter can be represented by a polytopic form; the polytopic coordinates are denotes $h_i(z_k), v_{\hbar}(z_k)$, and vary within the convex sets Ω_1 and Ω_2, respectively:

$$\Omega_1 = \left\{ h_i(z_k) \in \Re^r; h_i(z_k) = [h_1(z_k), \ldots \ldots ., h_r(z_k)]^T; h_i(z_k) \geq 0; \sum_{i=1}^{r} h_i(z_k) = 1 \right\} \tag{4}$$

$$\Omega_2 = \left\{ v_{\hbar}(z_k) \in \Re^{r_e}; v_{\hbar}(z_k) = [v_1(z_k) \ldots \ldots ., v_{r_e}(z_k)]^T; v_{\hbar}(z_k) \geq 0; \sum_{\hbar=1}^{r_e} v_{\hbar}(z_k) = 1 \right\} \tag{5}$$

The uncertain matrices, $\Delta A_i \in \Re^{n_x \times n_x}, \Delta B_i \in \Re^{n_x \times n_u}, \delta E_{\hbar} \in \Re^{n_x \times n_x}$, corresponding to the *i-th* and *k-th* sub-system contains the bounded uncertain terms which can be rewritten as: $\delta A_i(k) = H_{ai}\mathfrak{D}_{ai}(k)N_{ai}$; $\delta B_i(k) = H_{bi}\mathfrak{D}_{bi}(k)N_{bi}$ and $\delta E_{\hbar} = H_{e\hbar}\mathfrak{D}_{e\hbar}(k)N_{e\hbar}$, with $H_{ai}, H_{bi}, H_{ek}, N_{ai}, N_{bi}$, and $N_{e\hbar}$ are known constant matrices and $\mathfrak{D}_{ai}(k), \mathfrak{D}_{bi}(k)$, and $\mathfrak{D}_{ek}(k)$ are unknown matrices functions bounded, for all index $\varepsilon = a, b$ or e and $\theta = i, i \in I_r$ and $\hbar \in I_{r_e}$, one has $\mathfrak{D}_{\varepsilon\theta}^T(k).\mathfrak{D}_{\varepsilon\theta}(k) \leq I$.

Remark 1. The proposed approach relies on a well-known descriptor formulation which is well known to avoid the coupling terms between the feedback gains and the Lyapunov matrices. As a consequence, the number of LMIs decreases and relaxed conditions are obtained.

Remark 2. The sector nonlinearity approach[29,49] is used in this article to derive an exact polytopic form of general NLPV descriptor systems. This method can deal with a larger class of parametric dependencies system than, for example, linear, affine, or rational one. In References 23, 28, 39, for input-saturated control of nonlinear systems, this approach is exploited to represent the saturated actuators under a T-S form. In this work, the polytopic writing from an NLPV representation can generally be written equivalently, on a compact set of the state space as a T-S form, and a T-S system can also be seen as an NLPV system. Indeed, if the parameters vary in a compact set, it is possible in a direct and systematic way to rewrite these models in T-S form. Controller synthesis exclusively uses the polytopic vertices/sub-models, and only the convexity properties of the MFs are retained. The advantage of this method is that no approximation errors are introduced and the number of local models is reduced, this reduction allows us to decrease the number of constraints (relating to stability and stabilization), which increases the chances of finding a solution.

To ease the presentations, we assume that: $\mathbb{E} = (E_{\hbar} + \delta E_{\hbar})$, $\mathbb{A} = (A_i + \delta A_i)$, $\mathbb{B} = (B_i + \delta B_i)$.

In this article, the following assumptions will be considered for the output feedback controller design of uncertain NLPV descriptor system (1):

Assumption 1. The control input vector $u_{k(l)}$ is subject to symmetric amplitude limitation, that is:

$$-u_{\max(l)} \leq u_{k(l)} \leq u_{\max(l)}; l \in I_{n_l}. \tag{6}$$

where the control bound $u_{\max(l)} > 0$ is given, and l represents the lth component of input control.

Loss of performance or even instability may occur, when state trajectories evolve outside the domain of validity of closed-loop system. In such situations, one of the purposes of the present work is to handle the domain of validity of NLPV descriptor model into the controller synthesis stage, to ensure local closed-loop stability. In addition, due to the saturation nonlinearity effect, stability analysis required to be handled in local context. Besides the control input limitations, the system states are also bounded in engineering applications due to physical and/or safety reasons. The equivalent NLPV representation of a general nonlinear system obtained with the sector nonlinearity approach[49] is generally valid within a specific bounded set.[22] This domain of validity can be represented by a polyhedral set $\mathcal{D}_x$.

Assumption 2. The state trajectories of NLPV descriptor system are contained within the following polyhedral set (validity domain), $\mathcal{D}_x \in \Re^{n_x \times n_x}$ defined as follows:

$$\mathcal{D}_x = \left\{ x_k \in \Re^{n_x} : \mathcal{N}_m^T x_k \leq 1, m \in I_{n_q} \right\} \tag{7}$$

where the given matrix $\mathcal{N}_m \in \Re^{n_x}$ represents the state constraints of system (1). Moreover, the control input is subject to component wise saturation map: $sat(.) : \Re^{n_u} \rightarrow \Re^{n_u}$, defined as:

$$sat\left(u_{k(l)}\right) = sign\left(u_{k(l)}\right) \min\left(\left|u_{k(l)}\right|, u_{\max(l)}\right) \tag{8}$$

Here, we have slightly abused the notation by using $sat(.)$ to denote both the scalar valued and the vector valued saturation functions. Then, the standard saturation function is written as:

$$\begin{cases} sat_l\left(u_{k(l)}\right) = \left[sat_1\left(u_{k(1)}\right) \ \ldots \ \ldots \ sat_l\left(u_{k(l)}\right) \ \ldots \ \ldots \ sat_{n_u}\left(u_{k(n_u)}\right)\right] \\ sat_l\left(u_{k(l)}\right) = sign\left(u_{k(l)}\right) min\left(\left|u_{k(l)}\right|, u_{\max(l)}\right) \end{cases} \tag{9}$$

and:

$$sat\left(u_{k(l)}\right) = \begin{cases} u_{k(l)} & \text{if} \quad \left|u_{k(l)}\right| \leq u_{\max(l)} \\ u_{\max(l)} & \text{if} \quad \left|u_{k(l)}\right| > u_{\max(l)} \end{cases} \tag{10}$$

For $l \in I_{n_l}$, $u_{\max(l)} > 0$ represents the maximal saturation level of the lth control component, so:

$$sat\left(u_{k(l)}\right) = u_{k(l)} - \psi\left(u_{k(l)}\right) \tag{11}$$

where $\psi(u_{k(l)})$ represent the dead-zone nonlinearity, where $\psi(u_{k(l)}) = [\psi(u_{k(1)}) \ \ldots \ \ldots \ \psi(u_{k(l)})]^T$.

The lth component of decentralized dead-zone nonlinearity $\psi(u_{k(l)})$ is defined as:

$$\psi\left(u_{k(l)}\right) = \begin{cases} 0 & \text{if} \quad \left|u_{k(l)}\right| \leq u_{\max(l)} \\ u_{k(l)} - sign\left(u_{k(l)}\right).u_{\max(l)} & \text{if} \quad \left|u_{k(l)}\right| > u_{\max(l)} \end{cases} \tag{12}$$

Assumption 3. The disturbance signal ω_k is assumed to belong to the following class of function:

$$\mathcal{W}_\delta^2 = \left\{ \omega_k : \Re^+ \longrightarrow \Re^{n_\omega}, \sum_{k=1}^{\infty} \omega_k^T \omega_k \leq \delta, \forall k > 0, \delta > 0 \right\} \tag{13}$$

where the bound δ is known.

Using Finsler's Lemma, a better result have been proposed making it possible to decouple the PD matrices gains of control law with the Lyapunov matrix system, as well as to introduce free decision variables, helping to reduce the conservatism, and with progressively more relaxed results via controllers with nested convex sum, so:

Finsler's Lemma (50). Let $\xi_k \in \Re^{n_x}$, $\Phi = \Phi^T \in \Re^{n_x \times n_x}$ and $R \in \Re^{n_x}$ such that: $ank(R) \leq n_x$; the following expressions are equivalent:

$$\begin{cases} a)\ \xi_k^T \Phi \xi_k < 0 & : \quad \{\forall \xi_k \in \mathrm{R}^{n_\xi}; \xi_k \neq 0; R\xi_k = 0\} \\ b)\ \exists\ \mathcal{M} \in \mathrm{R}^{n_x} & : \quad \left\{\Phi + \mathcal{M}R + R^T \mathcal{M}^T < 0\right\} \end{cases} \tag{14}$$

In order to obtain less conservative stabilization conditions, we introduce free matrices to reduce the conservatism of the conditions expressed from double or triple sums.[51] In this work, we opted for the relaxation lemma of Reference 50, which constitutes a good compromise between complexity and reduction of conservatism, this relaxation scheme is presented as follows:

Lemma 1. (30) Let $\mathcal{T}_{ij}^{\hbar}(z_k)$ be symmetric matrix of appropriate dimensions, and h_i, $v_{\hbar}$, $(i,j) \in I_r \times I_r$ $\hbar \in I_{r_e}$ be any families of functions satisfying the property of convex sum.

The condition: $\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{\hbar=1}^{r_e} h_i h_j v_{\hbar} \mathcal{T}_{ij}^{\hbar}(z_k) < 0$, holds if:

$$\begin{cases} \mathcal{T}_{ii}^{\hbar}(z_k) < 0 \\ \frac{2}{r-1}\mathcal{T}_{ii}^{\hbar}(z_k) + \mathcal{T}_{ij}^{\hbar}(z_k) + \mathcal{T}_{ji}^{\hbar}(z_k) < 0 \end{cases} \qquad (i,j) \in I_r \times I_r, \hbar \in I_{r_e} \text{ and } i \neq j \tag{15}$$

Remark 3. In the framework of polytopic systems, many control design conditions can be presented in the following form:

$$\mathcal{T}_{hh}(z_k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z_k)\, h_j(z_k)\ \mathcal{T}_{ii} < 0 \tag{16a}$$

$$\mathcal{T}_{hh}^{\hbar}(z_k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z_k)\, h_j(z_k) \sum_{\hbar=1}^{r_e} \mathcal{T}_{ii}^{\hbar} < 0 \tag{16b}$$

$\mathcal{T}$ is a symmetric matrix of appropriate dimension, are linearly dependent on varying parameter $z_k \in \mathfrak{D}$, to convert PD condition $\mathcal{T}_{hh}(z_k)$ and $\mathcal{T}_{hh}^{\hbar}(z_k)$ into a finite set of LMIs while avoiding excessive computational burden of parameter-gridding algorithms. Without involving slack variables, the relaxation lemma mentioned in (15) leads to a good tradeoff between numerical complexity and conservatism.

Lemma 2. (29) Let $\mathfrak{X} = \mathfrak{X}^T > 0$, and $\mathfrak{Y}$ matrices of the appropriate dimensions, the following expressions holds:

$$(\mathfrak{Y} - \mathfrak{X})^T \mathfrak{X}^{-1} (\mathfrak{Y} - \mathfrak{X}) \geq 0 \leftrightarrow \mathfrak{Y}^T \mathfrak{X}^{-1} \mathfrak{Y} \geq \mathfrak{Y} + \mathfrak{Y}^T - \mathfrak{X}. \tag{17}$$

Lemma 3. (14) Let us consider $\mathfrak{X}$ and $\mathfrak{Y}$, and $\mathfrak{D}(k)$ be real matrices with appropriate dimensions, where $\mathfrak{D}(k)$ satisfying: $\mathfrak{D}^T(k)\,\mathfrak{D}(k) \leq I$, one has:

$$\begin{cases} \mathcal{H}(\mathfrak{X}\mathfrak{D}(k)\,\mathfrak{Y}) < 0; \forall \mathfrak{D}(k) \\ \mathcal{H}(\mathfrak{X}\mathfrak{D}(k)\,\mathfrak{Y}) < \mathfrak{X}^T\mathfrak{Y} + \mathfrak{Y}^T\mathfrak{X} \end{cases} \tag{18}$$

And for any some scalar: $\Sigma > 0$, and $\lambda > 0$ such that:

$$\begin{cases} \mathfrak{X}^T\mathfrak{Y} + \mathfrak{Y}^T\mathfrak{X} \leq \mathfrak{X}^T\Sigma\mathfrak{X} + \mathfrak{Y}\Sigma^{-1}\mathfrak{Y}^T, \Sigma > 0 \\ \mathfrak{X}^T\mathfrak{Y} + \mathfrak{Y}^T\mathfrak{X} \leq \lambda\mathfrak{X}^T\mathfrak{X} + \lambda^{-1}\mathfrak{Y}\mathfrak{Y}^T, \lambda > 0 \end{cases} \tag{19}$$

SOFC and DOFC can be designed using only the available information of the output vector/measured signal. In the following section, the controllers are derived using the descriptor approach. Sufficient LMI constraints are derived from Lyapunov's theory. Compared with Reference 28 in the following section, both SOFC and DOFC are proposed.

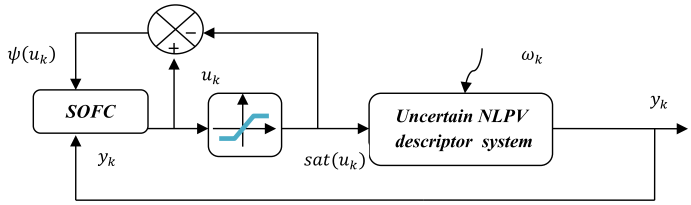

FIGURE 1 The proposed SOFC constrained controller design scheme [Colour figure can be viewed at wileyonlinelibrary.com]

3 | STATIC OUTPUT FEEDBACK CONTROLLER

3.1 | Controller design

The main objective is to synthesize a control law for the uncertain discrete-time NLPV descriptor system (1) subject to input saturation and external disturbances; the following unconstrained SOFC is envisaged:

$$u_k = F_{hv}(z_k) H_{hv}(z_k)^{-1} y_k \tag{20}$$

The PD gain matrices $F_{ij}(z_k) \in \mathfrak{R}^{n_u \times n_x}$ and $H_{ij}(z_k) \in \mathfrak{R}^{n_x \times n_x}$ for each $(i,j) \in I_r \times I_r$ are to be determined. The nonsingularity and the inverse of matrix $H_{hv}(z_k)$ in Equation (20) will be examined in Theorem 1.

In order to satisfy the desired performance, an SOFC under input saturation is designed as follows:

$$\begin{cases} \sum_{\hbar=1}^{re} v_{\hbar}(z_k)\mathbb{E}x_{k+} = \sum_{i=1}^{r} h_i(z_k)(\mathbb{A}x_k + \mathbb{B}sat(u_k)) + B_\omega \omega_k \\ y_k = Cx_k \\ u_k = F_{hv}(z_k) H_{hv}(z_k)^{-1} y_k \\ \psi(u_k) = u_k - sat(u_k) \end{cases} \tag{21a}$$

The proposed structure is reminiscent of dynamical anti-windup controllers for linear systems.[38]

Before the analysis of SOFC design problem, the corresponding closed-loop system should be formulated firstly with combining control law (20) with state-space equations (1). Therefore, the dynamic of closed-loop system is rewritten as:

$$\begin{cases} \sum_{\hbar=1}^{r_e} v_{\hbar}(z_k)\mathbb{E}x_{k+} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z_k) h_j(z_k)\left((\mathbb{A} + \mathbb{B}F_{ij}(z_k) H_{ij}\left(z_k\right)^{-1} C)x_k - \mathbb{B}\psi(u_k)\right) + B_\omega \omega_k \\ y_k = Cx_k \end{cases} \tag{21b}$$

The architecture of the proposed SOFC scheme is depicted in Figure 1.

Now in order to leave the quadratic framework proposed in Reference 15, we propose a new class of an adequate polytopic PD Lyapunov function to analyze stability conditions. Let us consider the polytopic PD nonquadratic Lyapunov function as follows:

$$V(x_k, z_k) = x_k^T E_v^T P_h(z_k)^{-1} E_v x_k; P_h(z_k) \in \mathfrak{R}^{n_x \times n_x}, P_h(z_k) = P_h(z_k)^T > 0 \tag{22}$$

where $P_h(z_k)$ and E_v are defined above.

Definition 1. (39,48) Let $P_h(z_k) \in \mathfrak{R}^{n_x \times n_x}$ be a symmetric PD Lyapunov matrix, and $E_v^T P_h(z_k)^{-1} E_v \geq 0$, let us define the ellipsoids (Lyapunov surfaces) as parameter dependent level set (PDLS) associated to $V(x_k, z_k)$ as follows:

$$\begin{cases} \mathcal{L}_v = \left\{x_k \in \mathfrak{R}^{n_x} : V(x_k, z_k) = x_k^T E_v^T P_h(z_k)^{-1} E_v x_k \leq \rho, \ \ \forall k > 0\right\} \\ \mathcal{L}_{v0} = \left\{x_k \in \mathfrak{R}^{n_x} : V(x_k, z_k) = x_k^T E_v^T P_h(z_k)^{-1} E_v x_k \leq 1, \ \ \text{for} \ \ k = 0\right\} \end{cases} \tag{23}$$

The PDLS is contractively invariant set; because the estimate of $\mathcal{L}v$ is the intersection of the ellipsoids, and is contained in the DoA which is nonconvex due to its associated nonquadratic Lyapunov function, and is included in $\mathcal{D}_x \cap \mathcal{D}_u$. PDLS based approaches can exploit finite bounds of the rates of the parameter variation and provide less conservative results than quadratic approaches assuming arbitrary parameter variation.[40] This will still lead to an estimate less conservative than the estimate based on the quadratic Lyapunov function.

Proposition 1. (17,22) A simple estimation of $\mathcal{L}_v$ and $\mathcal{L}_{v0}$ are the intersection of the ellipsoids, then:

$$\begin{cases} \mathcal{E}\left(E_v^T P_h(z_k)^{-1} E_v, \rho\right) = \bigcap_{i=1}^{r} \bigcap_{\hbar=1}^{r_e} \mathcal{E}\left(E_{\hbar}^T P_i^{-1} E_{\hbar}, \rho\right) \subseteq \mathcal{L}_v \\ \mathcal{E}\left(E_v^T P_h(z_k)^{-1} E_v, 1\right) = \bigcap_{i=1}^{r} \bigcap_{\hbar=1}^{r_e} \mathcal{E}\left(E_{\hbar}^T P_i^{-1} E_{\hbar}, 1\right) \subseteq \mathcal{L}_{v0} \end{cases} \tag{24}$$

where the PDLS is contractively invariant set, and is contained in the DoA.

Remark 4. The domain of validity $\mathcal{D}_x$ of NLPV descriptor system is obtained with the sector nonlinearity approach,[49] it can be described by the state constraints, taking this domain into account in the control design is essential to ensure both suitable closed-loop performance and stability, also $\mathcal{L}_v$ is a subset of the DoA included in the polyhedron set $\mathcal{D}_x$ of the closed-loop system related to $V(x_k, z_k)$, the set $\mathcal{D}_x$ represents the state-space region where trajectories are constrained and evolve within this set, due to both physical limitations and validity region of the NLPV descriptor model (1).

3.2 | LMI-based design conditions of constrained NLPV descriptor systems

In the light of the previous discussions, this work is concerned with proposing a systematic method to design a controller such that the closed-loop system satisfies the following properties, besides a sufficient condition to solve the following control problem:

Property 1. (Local stabilization) Then exist a positive definite function $V(x_k, z_k)$ such that all closed-loop trajectories starting from the set $\mathcal{L}_{v0}$, converge asymptotically to the origin in the absence of disturbance or the disturbances are vanishing ($\omega_k = 0$), for an arbitrary parameter z_k provided that the initial states belong to a specific set in the state-space. In the presence of disturbances satisfying Assumption 3, the controller guarantees that the trajectories of Equation (21b) are bounded.

Property 2. (Output to state stability and disturbances attenuation) Given vector $\mathcal{N}_m$ defined in Assumption 2 and a positive scalar δ depending in the type of disturbances involved in the dynamics of system (1). We distinguish two following control problems:

Control problem 1: when $\omega_k \neq 0$ and $\omega_k \in \mathcal{W}_\delta^2$. There exist positive scalar ρ and γ such that $\forall x_k \in \mathcal{L}_v \backslash \{0\}$, the corresponding closed-loop trajectory (21b) remains inside the validity domain $\mathcal{D}_x$ defined in Equation (7). Moreover, the $\mathfrak{L}_2$-gain of the output vector y_k is bounded as follows:

$$\|y_k\|_2^2 < \gamma^2 \|\omega_k\|_2^2 + \rho, \forall k > 0 \tag{25}$$

Control problem 2: Consider the NLPV descriptor model (1) design a nonlinear controller (20) such that: $\mathcal{L}_v \subseteq \mathcal{D}_x \cap \mathcal{D}_u$ is as large as possible, and is a contractively invariant set with respect to the closed-loop system (21) with $z_k \in \mathfrak{Q}$.

Assumption 4. $\mathcal{D}_{\mathcal{F}}\left(\mathcal{F}_{ij}, u_{max(l)}\right)$ denotes the polyhedral region associated with a matrix $\mathcal{F}_{ij} \in \mathfrak{R}^{n_u \times n_x}$ and a vector $u_{\max(l)} \in \mathfrak{R}^{n_u}$ defined by:

$$\mathcal{D}_{\mathcal{F}}\left(\mathcal{F}_{ij}, u_{max(l)}\right) = \left\{x_k \in \mathfrak{R}^{n_x} : |\mathcal{F}_{ij(l)} x_k| \leq u_{\max(l)}; \forall l \in I_{n_l}\right\} \tag{26}$$

$\mathcal{F}_{ij(l)}$ is a component of vector $\mathcal{F}_{ij}$ and $\mathcal{D}_{\mathcal{F}}\left(\mathcal{F}_{ij}, u_{max(l)}\right)$ is polyhedral set consisting of states for which saturation does not occur, and it is worth noticing that inside this admissible set the control input do not saturate and therefore, the evolution of the closed-loop system trajectories defined in Equation (21b) remains inside the validity domain $\mathcal{D}_x$ defined in Equation (7).

Remark 5. A certain degree of freedom is guaranteed when the system operates inside the region $\mathcal{D}_{\mathcal{F}}\left(\mathcal{F}_{ij}, u_{max(l)}\right)$.

Saturation Lemma (21). Given matrices, $H_{ij}(z_k) \in \Re^{n_x \times n_x}$ and $W_{ij}(z_k) \in \Re^{n_u \times n_x}$ for $(i, j) \in I_r \times I_r$, let us define the following set, where:

$$\mathcal{D}_u = \left\{ x_k \in \Re^{n_x} : |u_{k(l)} - v_{k(l)}| \le u_{\max(l)}; l \in I_{n_l} \right\} \tag{27}$$

If $x_k, u_{k(l)}$ and $v_{k(l)} \in \mathcal{D}_u$, then the inequality of the dead-zone nonlinearity $\psi(u_{k(l)})$, where $u_{k(l)}$ is defined in Equation (20) satisfy the following PD generalized sector condition:

$$\psi\left(u_{k(l)}\right)^T S_{ij(l)}(z_k)^{-1}\left[\psi\left(u_{k(l)}\right) - v_{k(l)}\right] \le 0 \tag{28}$$

Holds for any diagonal matrix $S_{ij} \in \Re^{n_u \times n_u}$, and for any scalar function: $h_i(z_k)$ satisfying the convex sum property, and $v_{k(l)} = (W_{ij}(z_k) H_{ij}(z_k)^{-1})_{(l)} x_k$.

Proof of Saturation Lemma. If $x_k \in \mathcal{D}_u$, it can be deduced:

$$-u_{\max(l)} \le u_{k(l)} - v_{k(l)} \le u_{\max(l)} \tag{29}$$

Notice that we have to show this:

$$\psi\left(u_{k(l)}\right) S_{ij(l)}(z_k)^{-1}\left[\psi\left(u_{k(l)}\right) - \left(W_{ij}(z_k) H_{ij}(z_k)^{-1}\right)_{(l)} x_k\right] \le 0; l \in I_{n_l} \tag{30}$$

To this end, three possible cases according to the value of $u_{k(l)}$ that may occur. It follows that: $u_{\max(l)} - u_{k(l)} + W_{ij}(z_k) H_{ij}(z_k)^{-1} x_k \ge 0$ and $-u_{\max(l)} - u_{k(l)} + W_{ij}(z_k) H_{ij}(z_k)^{-1} x_k \le 0$.

Case 1: $-u_{\max(l)} \le u_{k(l)} \le u_{\max(l)}$.
It follows that: $\psi(u_{k(l)}) = 0$ and $\psi(u_{k(l)}) S_{ij(l,l)}(z_k)^{-1}[\psi(u_{k(l)}) - (W_{ij}(z_k) H_{ij}(z_k)^{-1})_{(l)} x_k] = 0, \forall S_{ij} > 0$.
Case 2: $u_{k(l)} \ge u_{\max(l)}$, Then: $\psi(u_{k(l)}) = u_{k(l)} - u_{\max(l)} > 0$
and $u_{k(l)} - (W_{ij}(z_k) H_{ij}(z_k)^{-1})_{(l)} x_k \le u_{\max(l)}$
Hence, $\psi(u_{k(l)}) - (W_{ij}(z_k) H_{ij}(z_k)^{-1})_{(l)} x_k = u_{k(l)} - u_{\max(l)} + (W_{ij}(z_k) H_{ij}(z_k)^{-1})_{(l)} x_k \le 0$.
Since $\psi(u_{k(l)}) > 0$; and $(u_{k(l)}) S_{ij(l,l)}(z_k)^{-1}[\psi(u_{k(l)}) - (W_{ij}(z_k) H_{ij}(z_k)^{-1})_{(l)} x_k] \ge 0$, provided that $S_{ij(l,l)} \ge 0$.
Case 3: $u_{k(l)} \le -u_{\max(l)}$,
then: $\psi(u_{k(l)}) = u_{k(l)} + u_{\max(l)} < 0$ and $u_{k(l)} - (W_{ij}(z_k) H_{ij}(z_k)^{-1})_{(l)} x_k \ge -u_{\max(l)}$.
Combining the fact that: $\psi(u_{k(l)}) < 0$ and:
$\psi(u_{k(l)}) - (W_{ij}(z_k) H_{ij}(z_k)^{-1})_{(l)} x_k = u_{k(l)} - (W_{ij}(z_k) H_{ij}(z_k)^{-1})_{(l)} x_k + u_{\max(l)} \ge 0$.
Result in: $\psi(u_{k(l)}) S_{ij(l,l)}(z_k)^{-1}[\psi(u_{k(l)}) - (W_{ij}(z_k) H_{ij}(z_k)^{-1})_{(l)} x_k] \le 0$, for $S_{ij(l,l)} \ge 0$.

■

Remark 6. Saturation lemma plays a key role in decreasing the conservatism, it is motivated by the results of PD version of a *modified sector condition*,[44] this new version is especially appropriate for SOFC/DOFC context, and presents a tool to deal with the dead-zone nonlinearity $\psi(u_{k(l)})$, and this powerful tool provides a local characterization of NLPV descriptor system (1) by means of an extension of the absolute stability theory.

Inside the region $\mathcal{D}_{\mathcal{F}}\left(\mathcal{F}_{ij}, u_{max(l)}\right)$, the control input does not saturate, and therefore the evolution of the closed-loop system is described by the following model:

$$\begin{cases} \sum_{\hbar=1}^{r_e} v_{\hbar}(z_k)\, \mathbb{E} x_{k+} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z_k)\, h_j(z_k)\left(\mathbb{A}(z_k) + \mathbb{B}(z_k) F_{ij}(z_k) H_{ij}\left(z_k\right)^{-1} C) x_k\right) + B_\omega \omega_k \\ y_k = C x_k \end{cases} \tag{31}$$

remain inside the domain $\mathcal{D}_x$. Nevertheless outside the region $\mathcal{D}_{\mathcal{F}}\left(\mathcal{F}_{ij}, u_{max(l)}\right)$, the control input saturate and the stability of the saturated system must be analyzed.

In this section, an SOFC is designed; a solution of the stability analysis for uncertain and saturated discrete-time NLPV descriptor system with additive disturbances is given using a polytopic PD nonquadratic Lyapunov function. In order to obtain relaxed LMI constraints, the saturation bounds are taken into account. In the other hand, the estimation of the DoA is obtained by considering a bounded invariant set of initial system states, included in the validity domain $\mathcal{D}_x$. The following theorem summarized the obtained results:

Theorem 1. For a given the discrete-time uncertain NLPV descriptor system (1) with a parameter $z_k \in \mathfrak{D}$, under input saturation defined as (8), with the proposed controller (20), whose validity domain is defined by $\mathcal{D}_x$, is locally asymptotically stable if there exist a gain matrices: $P_i = P_i^T > 0$, where $P_i \in \Re^{n_x \times n_x}$; $X_i = X_i^T > 0$, where $X_i \in \Re^{n_x \times n_x}$ and $X_i = P_i^{-1}$; $W_{ij} \in \Re^{n_u \times n_x}$; $F_{ij} \in \Re^{n_u \times n_x}$; $H_{ij} \in \Re^{n_x \times n_x}$, for any positive diagonal gain matrix $S_{ij} \in \Re^{n_u \times n_u}$, $(i,j) \in I_r \times I_r$, a positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \gamma = \sqrt{\overline{\gamma}}$; $(\gamma^2 = \overline{\gamma})$, ρ and δ satisfying condition (15), such that the following inequalities holds:

$$\begin{cases} min\ \gamma \\ E_{\hbar}^T X_i E_{\hbar} > 0 \end{cases} \tag{32}$$

$$\rho + \overline{\gamma}\delta < 1 \tag{33}$$

$$\begin{bmatrix} -E_{\hbar}^T X_i E_{\hbar} & * \\ \mathcal{N}_m^T & -\frac{1}{\rho} \end{bmatrix} < 0 \tag{34}$$

$$\begin{bmatrix} -E_{\hbar} H_{ij} - H_{ij}^T E_{\hbar}^T + P_i & * \\ F_{ij(l)} - W_{ij(l)} & \frac{-\left(u_{(l)}^{max}\right)^2}{\rho} \end{bmatrix} < 0 \tag{35}$$

$$\mathcal{T}_{ij}^{\hbar} = \begin{bmatrix} \Pi_{ij}^{\hbar} & * \\ \Upsilon_{ij} & \zeta \end{bmatrix} < 0 \tag{36a}$$

$$\Pi_{ij}^{\hbar} = \begin{bmatrix} -E_{\hbar} H_{ij} - H_{ij}^T E_{\hbar}^T + P_i & * & * & * & * \\ C H_{ij} & -I & * & * & * \\ \left(A_i H_{ij} + B_i F_{ij} C\right) & 0 & \begin{matrix} -P_i \\ \left(\varepsilon_1 H_{ai} H_{ai}^T + (\varepsilon_1 + \varepsilon_3) H_{bi} H_{bi}^T + \varepsilon_2 H_{e\hbar} H_{e\hbar}^T\right) \end{matrix} & * & * \\ W_{ij} & 0 & -S_{ij}^T B_i{}^T & -2S_{ij}^T & * \\ 0 & 0 & B_{\omega}{}^T & 0 & -\overline{\gamma} I \end{bmatrix} \tag{36b}$$

$$\Upsilon_{ij} = \begin{bmatrix} N_{ai} H_{ij} & 0 & 0 & 0 & 0 \\ N_{bi} F_{ij} C & 0 & 0 & 0 & 0 \\ 0 & 0 & N_{e\hbar} P_i & 0 & 0 \\ 0 & 0 & 0 & N_{bi} S_{ij} & 0 \end{bmatrix} \tag{36c}$$

$$\zeta = \begin{bmatrix} -\varepsilon_1 & * & * & * \\ 0 & -\varepsilon_1 & * & * \\ 0 & 0 & -\varepsilon_2 & * \\ 0 & 0 & 0 & -\varepsilon_3 \end{bmatrix} \tag{36d}$$

Then, the SOFC (20) solves the control problem defined for the closed-loop NLPV descriptor system (21b).

Proof. Note that inequality (36.b) implies that: $H_{ij}^T E_{\hbar}^T P_i^{-1} E_{\hbar} H_{ij} \geq E_{\hbar} H_{ij} + H_{ij}^T E_{\hbar}^T - P_i$.

The same set of variables x_k can be represented as the feasible set of different LMIs. For instance, P_i^{-1} is positive definite matrix and $P_i^{-1} = P_i^{-T}$, then P_i^{-1} subject to a congruence transformation, is also positive definite:

$P_i^{-1} > 0 \leftrightarrow E_{\hbar}^T P_i^{-1} E_{\hbar} > 0, \forall E_{\hbar} \neq 0$
$E_{\hbar}^T P_i^{-1} E_{\hbar} \leftrightarrow H_{ij}^T E_{\hbar}^T P_i^{-1} E_{\hbar} H_{ij}; \forall H_{ij} \neq 0$, $E_{\hbar}$ is nonsingular.
$P_i^{-1} \leftrightarrow H_{ij}^T E_{\hbar}^T P_i^{-1} E_{\hbar} H_{ij}$

Also: $P_i^{-1} > 0 \leftrightarrow H_{ij}^T P_i^{-1} H_{ij} > 0$, $\forall H_{ij} \neq 0$ from Reference 29. The above inequality holds: $H_{ij}^T P_i^{-1} H_{ij} \geq H_{ij} + H_{ij}^T - P_i$, $P_i > 0$ then: $H_{ij} + H_{ij}^T > 0$, this guarantees that H_{ij} is nonsingular, there is the existence of H_{ij}^{-1}.

To study the local asymptotic stability of the closed-loop system (21b), the polytopic PD nonquadratic Lyapunov function defined in Equation (22) is considered, and a new LMI conditions for the design of SOFC to maintain stability performance with disturbance attenuation are designed. Specifically, the following definition will have to be satisfied for $x_k \in \mathcal{D}_x$, and $\forall x_k \in \mathcal{L}_v \backslash \{0\}$, it comes easily that any solution of the closed-loop system (21b), remains in the admissible set $\mathcal{D}_x$ defined in Equation (7) if:

$$\Delta V(x_k, z_k) + y_k^T y_k - \gamma^2 \omega_k^T \omega_k - 2\psi(u_k)^T S_{hv}(z_k)^{-1} \left[\psi(u_k) - W_{hv}(z_k) H_{hv}(z_k)^{-1} x_k\right] < 0 \tag{37}$$

With: $\Delta V(x_k, z_k) = V(x_{k+}, z_{k+}) - V(x_k, z_k)$ and $z_{k+} = z(k+1)$.

According to Equation (21b), we can give the following statement can be given by:

$$-\mathbb{E}(z_k) x_{k+} + \left(\mathbb{A}(z_k) + \mathbb{B}(z_k) F_{hv}(z_k) H_{hv}(z_k)^{-1} C\right) x_k - \mathbb{B}(z_k) \psi(u_k) + B_\omega \omega_k = 0 \tag{38}$$

This Equation (38) can be expressed by the following form:

$$\begin{bmatrix} \left(\mathbb{A}(z_k) + \mathbb{B}(z_k) F_{hv}(z_k) H_{hv}(z_k)^{-1} C\right) & -\mathbb{E}(z_k) & -\mathbb{B}(z_k) & B_\omega \end{bmatrix} \begin{bmatrix} x_k \\ x_{k+} \\ \psi(u_k) \\ \omega_k \end{bmatrix} = 0 \tag{39}$$

The inequality (37) can be written in the following form:

$$\begin{bmatrix} x_k \\ x_{k+} \\ \psi(u_k) \\ \omega_k \end{bmatrix}^T \begin{bmatrix} -E_v^T P_h(z_k)^{-1} E_v + C^T C & 0 & 0 & 0 \\ 0 & E_v^T P_h(z_k)^{-1} E_v & 0 & 0 \\ S_{hv}(z_k)^{-1} W_{hv}(z_k) H_{hv}(z_k)^{-1} & 0 & -2S_{hv}(z_k)^{-1} & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} x_k \\ x_{k+} \\ \psi(u_k) \\ \omega_k \end{bmatrix} < 0 \tag{40}$$

Via equality (39), inequality (40), and Finsler's lemma, $\mathcal{M}$ is chosen as follows: $\mathcal{M} = \begin{bmatrix} M & N & 0 & 0 \end{bmatrix}^T$, and we get:

$$\begin{bmatrix} -E_v^T P_h(z_k)^{-1} E_v + C^T C & 0 & 0 & 0 \\ 0 & E_v^T P_h(z_k)^{-1} E_v & 0 & 0 \\ S_{hv}(z_k)^{-1} W_{hv}(z_k) H_{hv}(z_k)^{-1} & 0 & -2S_{hv}(z_k)^{-1} & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} M \\ N \\ 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} \left(\mathbb{A}(z_k) + \mathbb{B}(z_k) F_{hv}(z_k) H_{hv}(z_k)^{-1} C\right) & -\mathbb{E}(z_k) & -\mathbb{B}(z_k) & B_\omega \end{bmatrix} + (*) < 0 \tag{41}$$

In order to obtain an LMIs problem, a good choice may consider $M = 0$ and $N = P_h(z_k)^{-1}$, and after some manipulations, we have:

$$\begin{bmatrix} -E_v^T P_h(z_k)^{-1} E_v + C^T C & * & 0 & 0 \\ N\left(\mathbb{A}(z_k) + \mathbb{B}(z_k) F_{hv}(z_k) H_{hv}(z_k)^{-1} C\right) & E_v^T P_h(z_k)^{-1} E_v - \mathcal{H}(N\mathbb{E}) & * & * \\ S_{hv}(z_k)^{-1} W_{hv}(z_k) H_{hv}(z_k)^{-1} & -\mathbb{B}(z_k)^T N^T & -2S_{hv}(z_k)^{-1} & 0 \\ 0 & B_\omega{}^T N^T & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{42}$$

Applying the congruence transformation property, pre and post multiplying (42) by *diag* $\begin{bmatrix} H_{hv}(z_k)^T & S_{hv}(z_k)^T & I & I \end{bmatrix}$ in the left hand-side and by *diag* $\begin{bmatrix} H_{hv}(z_k) & S_{hv}(z_k) & I & I \end{bmatrix}$ in the right hand-side, after some calculations the previous inequality leads to:

$$\Psi_{hv}^l + \delta\Psi_{hv}^l = \begin{bmatrix} -H_{hv}(z_k)^T E_v^T P_h(z_k)^{-1} E_v H_{hv}(z_k) + H_{hv}(z_k)^T C^T C H_{hv}(z_k) & * & 0 & 0 \\ (\mathbb{A}(z_k) H_{hv}(z_k) + \mathbb{B}(z_k) F_{hv}(z_k) C) & -P_h(z_k) & * & * \\ W_{hv}(z_k) & -S_{hv}(z_k)^T \mathbb{B}(z_k)^T & -2S_{hv}(z_k) & * \\ 0 & B_\omega{}^T & 0 & -\gamma^2 I \end{bmatrix} \tag{43}$$

with:

$$\boldsymbol{\Psi}_{hv}^{l}=\begin{bmatrix} -H_{hv}(z_k)^T E_v^T P_h(z_k)^{-1} E_v H_{hv}(z_k)+H_{hv}(z_k)^T C^T C H_{hv}(z_k) & * & 0 & 0\\ (\mathbb{A}(z_k) H_{hv}(z_k)+\mathbb{B}(z_k) F_{hv}(z_k) C)) & -P_h(z_k) & * & *\\ W_{hv}(z_k) & -S_{hv}(z_k)^T \mathbb{B}(z_k)^T & -2S_{hv}(z_k) & *\\ 0 & B_\omega{}^T & 0 & -\gamma^2 I \end{bmatrix} \tag{44}$$

$$\delta\boldsymbol{\Psi}_{hv}^{l}=\begin{bmatrix} 0 & \left(H_{hv}(z_k)^T \delta A_h(z_k)^T+C^T F_{hv}(z_k)^T \delta B_h^T\right) & 0 & 0\\ (\delta A_h(z_k) H_{hv}(z_k)+\delta B_h(z_k) F_{hv}(z_k) C) & -\delta E_v P_h(z_k)-P_h(z_k)^T \delta E_v^T & -\delta B_h S_{hv}(z_k) & 0\\ 0 & -S_{hv}(z_k)^T \delta B_h^T & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix}=\mathcal{S}+\mathcal{S}^T \tag{45}$$

with

$$\mathcal{S}=\begin{bmatrix} 0 & 0 & 0 & 0\\ (\delta A_h(z_k) H_{hv}(z_k)+\delta B_h(z_k) F_{hv}(z_k) C) & -\delta E_v P_h(z_k) & -\delta B_h S_{hv}(z_k) & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 & 0 & 0\\ \begin{matrix} H_{ai}\mathfrak{D}_{ai}(k) N_{ai} H_{hv}(z_k)+\\ H_{bi}\mathfrak{D}_{bi}(k) N_{bi} F_{hv}(z_k) C \end{matrix} & -H_{ei\hbar}\mathfrak{D}_{e\hbar}(k) N_{e\hbar} P_h(z_k) & -H_{bi}\mathfrak{D}_{bi}(k) N_{bi} S_{hv}(z_k) & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Then, applying condition (19), with $\Sigma=diag\,[\varepsilon_1\quad \varepsilon_2\quad \varepsilon_3\quad \varepsilon_4]$ and $\Sigma^{-1}=diag\left[\varepsilon_1^{-1}\quad \varepsilon_2^{-1}\quad \varepsilon_3^{-1}\quad \varepsilon_4^{-1}\right]$, we have:

$$\delta\boldsymbol{\Psi}_{hv}^{l}=\begin{bmatrix} \begin{pmatrix} \varepsilon_1^{-1}(N_{ai} H_{hv}(z_k) H_{hv}(z_k)^T N_{ai}^T+\\ N_{bi} F_{hv}(z_k) C C^T F_{hv}(z_k)^T N_{bi}^T) \end{pmatrix} & 0 & 0 & 0\\ 0 & \begin{pmatrix} \varepsilon_2^{-1}\left(N_{e\hbar} P_h(z_k) P_h(z_k)^T N_{e\hbar}^T\right)+\\ \left(\varepsilon_1 H_{ai} H_{ai}^T+(\varepsilon_1+\varepsilon_3) H_{bi} H_{bi}^T+\varepsilon_2 H_{e\hbar} H_{e\hbar}^T\right) \end{pmatrix} & 0 & 0\\ 0 & 0 & \varepsilon_3^{-1}\left(N_{bi} S_{hv}(z_k) S_{hv}(z_k)^T N_{bi}^T\right) & 0\\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then

$$\Psi_{hv}^{l}+\delta\Psi_{hv}^{l}=\begin{bmatrix} \chi_{hv}^{(1,1)} & * & 0 & 0\\ (\mathbb{A}(z_k) H_{hv}(z_k)+\mathbb{B}(z_k) F_{hv}(z_k) C) & \chi_h^{(2,2)} & * & *\\ W_{hv}(z_k) & -S_{hv}(z_k)^T \mathbb{B}(z_k)^T & \chi_{hv}^{(3,3)} & *\\ 0 & B_\omega{}^T & 0 & -\gamma^2 I \end{bmatrix}<0 \tag{46}$$

$$\begin{aligned} \chi_{hv}^{(1,1)}&=-H_{hv}(z_k)^T E_v^T P_h(z_k)^{-1} E_v H_{hv}(z_k)+H_{hv}(z_k)^T C^T C H_{hv}(z_k)\\ &\quad+\varepsilon_1^{-1}\left(N_{ai} H_{hv}(z_k) H_{hv}(z_k)^T N_{ai}^T+N_{bi} F_{hv}(z_k) C C^T F_{hv}(z_k)^T N_{bi}^T\right)\\ \chi_h^{(2,2)}&=-P_h(z_k)+\varepsilon_2^{-1}\left(N_{e\hbar} P_h(z_k) P_h(z_k)^T N_{e\hbar}^T\right)+\left(\varepsilon_1 H_{ai} H_{ai}^T+(\varepsilon_1+\varepsilon_3) H_{bi} H_{bi}^T+\varepsilon_2 H_{e\hbar} H_{e\hbar}^T\right)\\ \chi_{hv}^{(3,3)}&=-2S_{hv}(z_k)+\varepsilon_3^{-1}\left(N_{bi} S_{hv}(z_k) S_{hv}(z_k)^T N_{bi}^T\right) \end{aligned}$$

Applying the property of Schur complement[51] to dissociate this terms $\{\varepsilon_1^{-1}(N_{ai}H_{hv}(z_k)\,H_{hv}(z_k)^T N_{ai}^T + N_{bi}F_{hv}(z_k)\,CC^T F_{hv}(z_k)^T N_{bi}^T)\}$, $\{\varepsilon_2^{-1}\left(N_{e\hbar}P_h(z_k)\,P_h(z_k)^T N_{e\hbar}^T\right)\}$, and $\{\varepsilon_3^{-1}\left(N_{bi}S_{hv}(z_k)\,S_{hv}(z_k)^T N_{bi}^T\right)\}$, and according to the notations defined in precedent section, the conditions of Theorem 1 hold.

In order to ensure that $\varepsilon\left(E_v^T P_h(z_k)^{-1}E_v, 1\right)$ is a subset of $\mathcal{D}_x$, we need to enforce an inequality condition. In this regard, the polytopic set of states $\mathcal{D}_x$ can be alternatively rewritten as:

$$-2 + \mathcal{N}_m^T x_k + x_k^T\,\mathcal{N}_m < 0 \tag{47a}$$

According to the polytopic PD nonquadartic lyapunov function with considering the disturbances, the ellipsoidal DoA $\mathcal{L}_v = \varepsilon\left(E_v^T P_h(z_k)^{-1}E_v, \rho\right)$ is always a subset of $\mathcal{D}_x$, if and only if the following inequality is satisfied:

$$-\frac{1}{\rho} + \mathcal{N}_m^T x_k + x_k^T\,\mathcal{N}_m - x_k^T E_\hbar^T P_i^{-1} E_\hbar x_k < 0 \tag{47b}$$

The above inequality can be rewritten as:

$$\begin{bmatrix} x_k \\ 1 \end{bmatrix}^T \begin{bmatrix} -E_\hbar^T P_i^{-1} E_\hbar & * \\ \mathcal{N}_m^T & -\frac{1}{\rho} \end{bmatrix} \begin{bmatrix} x_k \\ 1 \end{bmatrix} < 0 \leftrightarrow \begin{bmatrix} -E_\hbar^T P_i^{-1} E_\hbar & * \\ \mathcal{N}_m^T & -\frac{1}{\rho} \end{bmatrix} < 0 \tag{48}$$

Taking into account the ellipsoid: $\varepsilon\left(E_v^T P_h(z_k)^{-1}E_v, 1\right) \subseteq \varepsilon\left(E_v^T P_h(z_k)^{-1}E_v, \rho\right)$, and for all $x_k \in \mathcal{L}_v\backslash\{0\}$ which implies that $\mathcal{N}_m^T x_k < 1$, this proves the inclusion $\mathcal{L}_v \subseteq \mathcal{D}_x$. Theorem 1 gives an LMI conditions for the ellipsoid $\mathcal{E}\left(E_v^T P_h(z_k)^{-1}E_v, \rho\right)$, to be inside the validity domain $\mathcal{D}_x$, for the closed-loop system under SOFC, so $\mathcal{E}\left(E_v^T P_h(z_k)^{-1}E_v, 1\right)$ can solve an estimation of the DoA for the control system, if and only if $\mathcal{E}\left(E_v^T P_h(z_k)^{-1}E_v, 1\right) \subseteq \mathcal{D}_x$ and for all ellipsoids satisfying the set invariance condition of Theorem 1, we would choose the largest one to obtain the least conservative estimate of the DoA.

Inequality (35) implies clearly that: $-E_\hbar H_{ij} - H_{ij}^T E_\hbar^T > P_i > 0$. This in its turn implies that H_{ij} is regular and invertible; since: $-H_{ij}^T E_\hbar^T P_i^{-1} E_\hbar H_{ij} > -E_\hbar H_{ij} - H_{ij}^T E_\hbar^T + P_i$, inequality (35), yields:

$$\begin{bmatrix} -H_{ij}^T E_\hbar^T P_i^{-T} E_\hbar H_{ij} & * \\ F_{ij(l)} - W_{ij(l)} & \frac{-\left(u_{(l)}^{max}\right)^2}{\rho} \end{bmatrix} < 0 \tag{49}$$

We multiply (49) by: $diag[H_{ij}(z_k)^{-T}\ 1]$ in the left and by its transpose in the right-hand side, we get:

$$\begin{bmatrix} -E_\hbar^T P_i^{-T} E_\hbar & H_{ij}^{-T}\left(W_{ij(l)}^T - F_{ij(l)}^T\right) \\ \left(F_{ij(l)} - W_{ij(l)}\right) H_{ij}^{-1} & \frac{-\left(u_{(l)}^{max}\right)^2}{\rho} \end{bmatrix} < 0 \tag{50}$$

with: $\vartheta = (F_{ij(l)} - W_{ij(l)})H_{ij}(z_k)^{-1}$ and $P_i = P_i^T$.

Pre and post multiplying (50) by $diag\left[x_k^T\ \ 1\right]$, and its transpose, applying Schur complement,[51] with taking into consideration the $\mathfrak{L}_2$ gain defined in Equation (25), we have: $-x_k^T E_\hbar^T P_i^{-1} E_\hbar x_k + \frac{\rho}{u_{max}^2} x_k^T \vartheta^T \vartheta x_k \leq 0$: $\forall x_k \in \mathcal{L}_v\backslash\{0\}$, then; $x_k^T E_v^T P_h^{-T} E_v x_k \subseteq \varepsilon\left(E_v^T P_h(z_k)^{-1}E_v, \rho\right) \leq \rho$; $x_k^T \vartheta^T \vartheta x_k \leq \left(u_{(l)}^{max}\right)^2 \rightarrow |\vartheta x_k| \leq u_{(l)}^{max}$, from that we can conclude that: $\mathcal{L}_v \subseteq \mathcal{D}_u$.

Since $S^{-1} > 0$, by property of the Saturation Lemma, it follows that $\Delta V(x_k, z_k) < 0$, for $\forall x_k \in \mathcal{L}_v \subseteq \mathcal{D}_x \cap \mathcal{D}_u$, this guarantee that the origin of the closed-loop system (21b) is locally asymptotically stable and $\mathcal{L}_v \subseteq \mathcal{D}_x \cap \mathcal{D}_u$ is contractively invariant set with respect to the system (1), this completes the proof and proves control Problem 1. ■

Since $\mathcal{L}_v \subseteq \mathcal{D}_x \cap \mathcal{D}_u$, by the Saturation Lemma, it follows from (38) that:

$$\Delta V\left(x_k, z_k\right) + y_k^T y_k - \gamma^2 \omega_k^T \omega_k < 0 \tag{51}$$

From now, two cases can be distinguished:

Case 1: If $\omega_k = 0$, it follows that: $\Delta V(x_k, z_k) < 0, \forall x_k \subseteq \mathcal{E}\left(E_v^T P_h(z_k)^{-1} E_v, 1\right)$, where $\Delta V(x_k, z_k)$ is defined above. Consider that (51) ensures that: $\forall x(0) \in \mathcal{E}\left(E_v^T P_h(z_k)^{-1} E_v, 1\right)$, the corresponding closed-loop trajectories converge asymptotically to the origin.

Case 2: If $\omega_k \neq 0$, and $\omega_k \in \mathcal{W}_\delta^2$, summing both sides of inequality (51) from 0 to *k-tf* instant, yields: $V(x_{k+}, z_{k+}) - V(x_k, z_k) + y_k^T y_k - \gamma^2 \omega_k^T \omega_k < 0$, therefore *k-tf* $\rightarrow \infty$, we have:

$$V(x(\infty)) - V(x(0)) + \sum_{k=0}^{\infty} y_k^T y_k - \gamma^2 \sum_{k=0}^{\infty} \omega_k^T \omega_k < 0, \forall x_k \in \mathcal{L}_v \tag{52}$$

With the zero condition: $V(x(\infty)) \leq \rho$ and $V(x(0)) = 0$, and from (52), we obtain:

$$\sum_{k=0}^{\infty} y_k^T y_k < V(x(\infty)) - V(x(0)) + \gamma^2 \sum_{k=0}^{\infty} \omega_k^T \omega_k \tag{53}$$

$$\text{This latter is equivalent to}: \sum_{k=0}^{\infty} y_k^T y_k < \rho + \gamma^2 \sum_{k=0}^{\infty} \omega_k^T \omega_k < 1 \tag{54}$$

Which means that the $\mathfrak{L}_2$-gain of output signal y_k is bounded, that: $\|y_k\|_2^2 < \gamma^2 \|\omega_k\|_2^2 + \rho, \forall k > 0$. Then, the control problem 2 was solved.

Since local control context is considered for the design method in this article, it is therefore desirable to achieve the largest DoA in many cases, and the domain of attraction (DoA) in this article is nonconvex due to its associated nonquadratic Lyapunov function. The following optimization problem is proposed to achieve the control goal.

3.3 | Optimization of the domain of attraction

In this section, the design method will be formulated as a multi-objective optimization problem. Two objectives are considered: the first one is maximization of the disturbance rejection by augmenting the level amplitude/energy disturbances δ, and/or minimizing the $\mathfrak{L}_2$ attenuation level γ, the second is the estimation of the size of the DoA α. In fact, both objectives could be unified by minimizing $\alpha^{-2} + \gamma^2$.[39] The domain of initial condition $\mathcal{E}\left(E_v^T P_h(z_k)^{-1} E_v, 1\right)$ is included in the DoA $\mathcal{E}\left(E_v^T P_h(z_k)^{-1} E_v, \rho\right)$, we can maximize the latter by optimizing the set $\mathcal{E}\left(E_v^T P_h(z_k)^{-1} E_v, 1\right)$. In order to obtain a sufficiently large estimated DoA, two typical types of the reference shape $\mathcal{X}_\mathcal{R}$ are considered.

Consider an ellipsoid $\mathcal{X}_\mathcal{R} = \left\{x_k \in \mathfrak{R}^{n_x} : x_k^T \mathcal{R} x_k \leq 1\right\}$, where $\mathcal{X}_\mathcal{R} \subseteq \mathfrak{R}^{2n_x}$ is a prescribed bounded convex ellipsoid set containing the origin, and $\mathcal{R} > 0$ is the diagonal matrix with compatible dimension. The following optimization problem gives:

$$\begin{cases} a) \sup \alpha \\ b) \alpha \mathcal{X}_\mathcal{R} \subseteq \bigcap_{i=1}^{r} \bigcap_{\hbar=1}^{r_e} \mathcal{E}\left(E_\hbar^T P_i^{-1} E_\hbar, 1\right) \end{cases} \tag{55}$$

Constraint (b) is equivalent to: $\left(\alpha x_k^T\right) E_\hbar^T P_i^{-1} E_\hbar (\alpha x_k) \leq x_k^T \mathcal{R} x_k$ and $\alpha^{-2}\mathcal{R} - E_\hbar^T P_i^{-1} E_\hbar \geq 0$, setting $\alpha^{-2} = \eta$, the following LMIs constraints give sufficient condition:

$$\begin{bmatrix} -\eta \mathcal{R} & * \\ E_\hbar & -P_i \end{bmatrix} < 0 \tag{56}$$

In the other hand, consider a polyhedron set that: $\mathcal{X}_\mathcal{R} = co\left\{x_0^1, x_0^2, \ldots.., x_0^l\right\}$, with $x_0^1, x_0^2, \ldots.., x_0^l$, are a priori points given in $\mathfrak{R}^{n_x}$, or states for the vertex of the bounded polyhedral set $\mathcal{X}_\mathcal{R}$. Thus $\alpha \mathcal{X}_\mathcal{R} \subseteq \bigcap_{i=1}^{r} \bigcap_{\hbar=1}^{r_e} \mathcal{E}\left(E_\hbar^T P_i^{-1} E_\hbar, 1\right)$ is equivalent to: $\alpha\left(x_0^i\right)^T E_{\hbar}^T P_i^{-1} E_{\hbar}\alpha\left(x_0^i\right) \leq 1; \forall i \in I_r, \alpha^{-2}=\eta$, with using a Schur complement,[51] gives directly sufficient LMIs constraints:

$$\begin{bmatrix} -\eta I & * \\ E_{\hbar} x_0^i & -P_i \end{bmatrix} < 0 \tag{57}$$

Theorem 2. For the given the uncertain NLPV descriptor system (1) under input saturation defined as (8),with a parameter $z_k \in \mathfrak{D}$, and the proposed controller (21), whose validity domain is defined in $\mathcal{D}_x$, is locally asymptotically stable and the estimation of DoA is maximized, if there exist gain matrices: $P_i = P_i^T > 0$, where $P_i \in \mathfrak{R}^{n_x \times n_x}$; $W_{ij} \in \mathfrak{R}^{n_u \times n_x}$; $F_{ij} \in \mathfrak{R}^{n_u \times n_x}$; and $H_{ij} \in \mathfrak{R}^{n_x \times n_x}$, for any positive diagonal gain matrix $S_{ij} \in \mathfrak{R}^{n_u \times n_u}$, a positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \gamma, \rho, \delta$, and η satisfying condition (11), solution of the following optimization problem:

$$s.t.\begin{cases} \min\limits_{\mathcal{R}, P_i > 0} \eta \\ LMIs\ (29) \sim (36) \\ LMI\ (56)\ or\ (57) \end{cases} \tag{58}$$

Moreover, the attenuation level γ is guaranteed to be the minimum possible, and the DoA is maximized, with its size represented by α.

4 | DYNAMIC OUTPUT FEEDBACK CONTROLLER

4.1 | Control problem

The objective now is to design a stabilizing DOFC for the proposed constrained system. As previously, the development is obtained by representing the saturation function as a modified sector condition of dead-zone nonlinearity, and by solving an optimization problem under LMI constraints. Let us consider the following nth order DOFC defined by:

$$\begin{cases} x_F(k+1) = A_{hv}^F(z_k) P_{h2}^{-1}(z_k) x_F(k) + B_{hv}^F(z_k) P_{h1}^{-1}(z_k) y(k) \\ u_k = C_{hv}^F(z_k) P_{h2}^{-1}(z_k) x_F(k) + D_{hv}^F(z_k) P_{h1}^{-1}(z_k) y(k) \end{cases} \tag{59}$$

where $x_F(k) \in \mathfrak{R}^{n_x}$ is the controller state vector, $A_{hv}^F(z_k) \in \mathfrak{R}^{n_x \times n_x}$, $B_{hv}^F(z_k) \in \mathfrak{R}^{n_u \times n_x}$, $C_{hv}^F(z_k) \in \mathfrak{R}^{n_x \times n_x}$, $D_{hv}^F(z_k) \in \mathfrak{R}^{n_u \times n_x}$ are the PD controller gain matrices; $P_{h1}(z_k) \in \mathfrak{R}^{n_x \times n_x}$ and $P_{h2}(z_k) \in \mathfrak{R}^{n_x \times n_x}$ are PD symmetric matrices to be determined, to guarantee the stability of the uncertain discrete-time NLPV descriptor system (1).

Controller order (59) can be adapted according to the system dynamics of the control objectives, and then has to be designed to guarantee the local stability and some performance requirements for the closed-loop system. Because of the input limitation, the actual control signal injected into the system is subject to the saturation effect. The closed-loop system defined from combining (1) and (59) is written under the descriptor form, introducing the augmented state variable: $\overline{x}_k = \begin{bmatrix} x_k & x_F(k) \end{bmatrix}^T$, is described as follows:

$$\overline{E}\overline{x}(k+1) = \overline{A}\overline{x}(k) + \overline{B}\psi(u_k) + \overline{B}_\omega \omega_k \tag{60}$$

and the controller output is rewritten as:

$$u_k = \overline{K}_{hv}(z_k)\overline{x}_k \tag{61}$$

Then

$$\begin{cases} \overline{E}\overline{x}(k+1) = \overline{A}\overline{x}(k) + \overline{B}\psi(u_k) + \overline{B}_\omega \omega_k \\ y_k = \overline{C}\overline{x}_k \\ u_k = \overline{K}_{hv}(z_k)\overline{x}_k \\ \psi(u_k) = u_k - \mathrm{sat}(u_k) \end{cases} \tag{62}$$

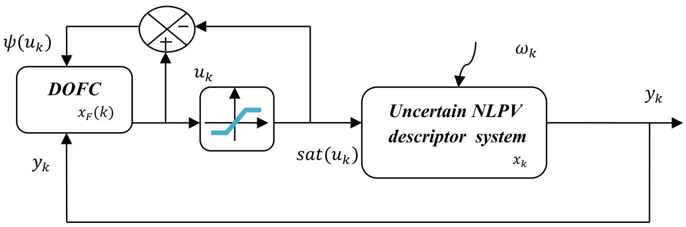

FIGURE 2 DOFC constrained controller design scheme [Colour figure can be viewed at wileyonlinelibrary.com]

with

$$\overline{E} = \begin{bmatrix} \mathbb{E}(z_k) & 0 \\ 0 & I \end{bmatrix}; \; \overline{A} = \begin{bmatrix} \mathbb{A}(z_k) + \mathbb{B}(z_k) D^F_{hv}(z_k) P^{-1}_{h1}(z_k) C & \mathbb{B}(z_k) C^F_{hv}(z_k) P^{-1}_{h2}(z_k) \\ B^F_{hv}(z_k) P^{-1}_{h1}(z_k) C & A^F_{hv}(z_k) P^{-1}_{h2}(z_k) \end{bmatrix}; \; \overline{B} = \begin{bmatrix} -\mathbb{B}(z_k) \\ 0 \end{bmatrix};$$

$$\overline{K}_{hv}(z_k) = \begin{bmatrix} D^F_{hv}(z_k) P^{-1}_{h1}(z_k) C & C^F_{hv}(z_k) P^{-1}_{h2}(z_k) \end{bmatrix}; \; \overline{W}_{hv} = \begin{bmatrix} W^1_{ij}(z_k) & W^2_{ij}(z_k) \end{bmatrix}; \; \overline{C} = \begin{bmatrix} C & 0 \end{bmatrix};$$

$$\overline{B}_\omega = \begin{bmatrix} B_\omega \\ 0 \end{bmatrix}; \mathbb{E}(z_k), \mathbb{A}(z_k), \mathbb{B}(z_k) \text{ are defined above.}$$

The architecture of the proposed DOFC conception is based on the scheme depicted in Figure 2.

4.2 | LMI-based design conditions of constrained NLPV system

Local stability performance specifications of the closed-loop system (62) will be presented in terms of Lyapunov analysis tools. Our goal is to propose a systematic method to design a DOFC, such that the closed-loop system satisfies the following properties and assumptions:

Assumption 4. (State constraint) For any initial augmented state: $\overline{x}(0) \in \overline{\mathcal{E}}\left(\overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v, 1\right) \subseteq \overline{\mathcal{E}}\left(\overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v, \rho\right)$ and for any disturbance signal ω_k belongs to $\mathcal{W}^2_\delta$, the trajectories of the closed-loop system (63) remains in the polyhedral region described by this linear inequalities:

$$\bar{\mathcal{D}}_{\overline{x}_k} = \left\{ \overline{x}_k \in \mathfrak{R}^{2n_x} \; : \; \bar{\mathcal{N}}^T_m \overline{x}_k \le 1; \forall k \in I_r; m \in I_{n_q} \right\} \tag{63}$$

where $\bar{\mathcal{N}}_m = \begin{bmatrix} \mathcal{N}_m & 0 \end{bmatrix}^T$; implying that the system state x_k remains in the validity domain $\mathcal{D}_x$ of system (1).

Assumption 5. Let us define the set $\mathcal{D}_{\bar{k}}\left(\mathcal{K}_{ij}, u_{max(l)}\right)$ which denotes the polyhedral region associated with a matrix $\mathcal{K}_{ij} \in \mathfrak{R}^{n_u \times n_x}$ and a vector $u_{\max(l)} \in \mathfrak{R}^{n_u}$ defined by:

$$\mathcal{D}_{\bar{k}}\left(\mathcal{K}_{ij}, u_{max(l)}\right) = \left\{ \overline{x}_k \in \mathfrak{R}^{2n_x} \; : \; |\bar{\mathcal{K}}_{ij(l)} x_k| \le u_{\max(l)}; \forall l \in I_{n_l} \right\} \tag{64}$$

$\mathcal{K}_{ij(l)}$ is a component of vector $\mathcal{K}_{ij}$ and $\mathcal{D}_{\bar{k}}\left(\mathcal{K}_{ij}, u_{max(l)}\right)$ is polyhedral set consisting of states, for which saturation does not occur, it is worth noticing that inside this admissible set the control input do not saturate and therefore, the evolution of the closed-loop system trajectories defined in Equation (62) remains inside the domain of validity $\mathcal{D}_{x_k}$ defined in Equation (64).

A certain degree of freedom is guaranteed when the system operates inside the region $\mathcal{D}_{\mathcal{K}}\left(\mathcal{K}_{ij}, u_{max(l)}\right)$.

Property 3. (Finite $\mathfrak{L}_2$-norm performance) For any $\omega_k \in \mathcal{W}^2_\delta$ such that no saturation occur, there exists a positive real number γ such that: $\|y_k\|_2^2 < \gamma^2 \|\omega_k\|_2^2 + \rho$, for all $k > 0$. This inequality is provided to attenuate the effect of ω_k on the output of system y_k, where γ is the attenuation level to be determined.

The proof of the $\mathfrak{L}_2$-*norm performance* is similar of the case of SOFC.

The main goal now is to provide LMIs stability conditions allowing to find PD gain matrices: $A^F_{hv}(z_k)$, $B^F_{hv}(z_k)$, $C^F_{hv}(z_k)$, $D^F_{hv}(z_k)$, $P_{h1}(z_k)$, and $P_{h2}(z_k)$ ensuring the stability of Equation (62) under DOFC, then some important lemmas results are needed for design problem in Section 4.1 will be presented, is based on the use of the sector bounded condition from Reference 24, then:

Saturation Lemma (Sector bound condition for input saturation). Let us consider: $\overline{W}_{ij}(z_k) \in \Re^{n_u \times 2n_x}$, $\overline{K}_{ij} \in \Re^{n_u \times 2n_x}$, with $\overline{W}_{ij}(z_k) = \left[W^1_{ij}(z_k) \quad W^2_{ij}(z_k)\right]$ and $\overline{K}_{ij}(z_k) = \left[D^F_{ij}(z_k) P^{-1}_{i1}(z_k) C \quad C^F_{ij}(z_k) P^{-1}_{i2}(z_k)\right]$; $W^1_{ij}(z_k) \in \Re^{n_u \times n_x}$; $W^2_{ij}(z_k) \in \Re^{n_u \times n_x}$; $(i,j) \in I_r \times I_r$, we define the following polyhedral set $\bar{\mathcal{D}}_{u_k}$ as follows:

$$\bar{\mathcal{D}}_{u_k} = \left\{ \overline{x}_k \in \Re^{2n_x} : \left| \left(\overline{K}_{ij(l)}(z_k) - \overline{W}_{ij(l)}(z_k) \right)_{(l)} \overline{x}_k \right| \leq u_{\max(l)} \right\} \tag{65}$$

Consider the function: $\psi(u_k)_{(l)} = u_{k(l)} - sat(u_{k(l)})$, with $u_{k(l)}$ defined in Equation (61):
If $\overline{x}_k \in \Re^{2n_x}$ and $\overline{x}_k \in \bar{\mathcal{D}}_{x_k} \cap \bar{\mathcal{D}}_{u_k}$, then the following condition is verified:

$$\psi^T\left(u_{k(l)}\right) S_{ij(l)}(z_k)^{-1} \left[\psi(u_k) - \left(\overline{W}_{ij}(z_k)\overline{x}_k\right)_{(l)}\right] \leq 0 \tag{66}$$

For any positive diagonal matrix $S_{ij(l)} \in \Re^{n_u \times n_u}$ is a positive diagonal matrix, and for any scalar function $h_i(z_k)$, $\in I_r$, satisfying the convex sum property.

Proof. Assume that $\overline{x}_k \in \bar{\mathcal{D}}_{x_k}$ it follows that $\overline{x}_k \in \bar{\mathcal{D}}_{u_k}$, since scalar function $h_i(z_k)$ $\forall i \in I_r$ satisfy the convex sum property, hence it follows that:

$$-u_{\max(l)} \leq \left(\overline{K}_{ij}(z_k) - \overline{W}_{ij}(z_k)\right)_{(l)} \overline{x}_k \leq u_{\max(l)} \ \forall (i,j) \in I_r \times I_r,\ l \in I_{nl}$$

Let $S_{ij(l)} > 0$ is lth element of the ith row and jth column of diagonal matrix S.
Consider three cases as follow:

Case 1: $-u_{max(l)} \leq \left(\overline{K}_{ij}(z_k) - \overline{W}_{ij}(z_k)\right)_{(l)} \overline{x}_k \leq u_{max(l)}$, in this case $\psi(u_{k(l)}) = 0$, and then:

$$\psi^T\left(u_{k(l)}\right) S_{ij(l)}(z_k)^{-1} \left[\psi\left(u_{k(l)}\right) - \overline{W}_{ij}(z_k)\overline{x}_k\right] = 0.$$

Case 2: $u_{k(l)} > u_{max(l)}$, in this case $\psi(u_{k(l)}) = u_{k(l)} - u_{max(l)}$. If $\overline{x}_k \in \bar{\mathcal{D}}_{u_k}$, it follows that:
$\overline{K}_{hv}(z_k)\overline{x}_k - \overline{W}_{ij}(z_k)\overline{x}_k \leq u_{max(l)}$, hence it follows that:

$$\psi\left(u_{k(l)}\right) - \overline{W}_{ij}(z_k)\overline{x}_k = \left(\overline{K}_{ij}(z_k) - \overline{W}_{ij}(z_k)\right)\overline{x}_k - u_{max(l)} \leq 0$$

And since in this case: $\psi(u_{k(l)}) > 0$, one gets: $\psi^T\left(u_{k(l)}\right) S_{ij(l)}(z_k)^{-1} \left[\psi\left(u_{k(l)}\right) - \overline{W}_{ij}(z_k)\overline{x}_k\right] \geq 0, \quad \forall S_{ij(l)} > 0.$
Case 3: $u_{k(l)} < -u_{max(l)}$. In this case $\psi(u_{k(l)}) = u_{k(l)} + u_{max(l)}$. If $\overline{x}_k \in \bar{\mathcal{D}}_{u_k}$, if that:

$$\psi\left(u_{k(l)}\right) - \overline{W}_{ij}(z_k)\overline{x}_k = \left(\overline{K}_{ij}(z_k) - \overline{W}_{ij}(z_k)\right)\overline{x}_k + u_{\max(l)} \geq 0.$$

And since in this case: $\psi(u_{k(l)}) < 0$, one gets: $\psi^T\left(u_{k(l)}\right) S_{ij(l)}(z_k)^{-1} \left[\psi(u_k(l)) - \overline{W}_{ij}(z_k)\overline{x}_k\right] \leq 0, \quad \forall S_{ij(l)} > 0$
From these three cases, provided that $\overline{x}_k \in \bar{\mathcal{D}}_{u_k}$, we can conclude that:
$\psi^T\left(u_{k(l)}\right) S_{ij(l)}(z_k)^{-1} \left[\psi(u_k(l)) - \overline{W}_{ij}(z_k)\overline{x}_k\right] \leq 0$ from where follows (66). ∎

Lemma 5. The ellipsoid $\mathcal{E}\left(\overline{E}_v^T \overline{P}_h(z_k)^{-1}\overline{E}_v, 1\right) \subseteq \mathcal{E}\left(\overline{E}_v^T \overline{P}_h(z_k)^{-1}\overline{E}_v, \rho\right)$ is included in the polyhedral set $\bar{\mathcal{D}}_{\overline{x}_k}$ defined in (64) if and only if:

$$\text{If } \omega_k = 0 \begin{bmatrix} -E_{\hbar}^T X_{i1} E_{\hbar} & * & * \\ 0 & -X_{i2} & * \\ \mathcal{N}_m^T & 0 & -1 \end{bmatrix} < 0; \forall m \in I_q \tag{67a}$$

$$\text{If } \omega_k \neq 0 \begin{bmatrix} -E_{\hbar}^T X_{i1} E_{\hbar} & * & * \\ 0 & -X_{i2} & * \\ \mathcal{N}_m^T & 0 & -\frac{1}{\rho} \end{bmatrix} < 0; \forall m \in I_q \tag{67b}$$

Lemma 6. The ellipsoid $\overline{\mathcal{E}}\left(\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v, 1\right) \subseteq \overline{\mathcal{E}}\left(\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v, \rho\right)$ is included in the polyhedral set $\bar{\mathcal{D}}_{u_k}$ defined in Equation (64):

$$\text{if } \omega_k = 0\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)^T\left(\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v\right)^{-1}\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right) < u^2_{\max(l)} \tag{68a}$$

$$\text{If } \omega_k \neq 0\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)^T\left(\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v\right)^{-1}\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right) < \frac{u^2_{\max(l)}}{\rho} \tag{68b}$$

where $\overline{K}_{ij(l)}(z_k)$ and $\overline{W}_{ij(l)}(z_k)$ are, respectively, the lth rows of the gain matrices $\overline{K}_{ij}(z_k)$ and $\overline{W}_{ij}(z_k)$.

Proof. It is required to consider a control limit condition for the inputs $u_{k(l)}$ and $v_{k(l)}$, denoting the maximal available control norm as $u_{max(l)}$, it follows that:

$$\left\| \left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)\overline{x}_k \right\|_2^2 < u^2_{\max(l)} \tag{69}$$

we obtain:

$$\overline{x}_k^T\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)^T\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)\overline{x}_k < u^2_{\max(l)} \tag{70}$$

or equivalently:

$$\frac{\overline{x}_k^T\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)^T\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)\overline{x}_k}{u^2_{\max(l)}} < 1 \tag{71}$$

Since we are investing conditions and invariant ellipsoidal region satisfying: $\overline{x}_k^T\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v\overline{x}_k < \rho$, $(\omega_k \in \mathcal{W}_\delta^2)$, we can ensure that Equation (71) is satisfied by imposing (sufficient condition):

$$\frac{\overline{x}_k^T\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)^T\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)\overline{x}_k}{u^2_{\max(l)}} < \overline{x}_k^T\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v\overline{x}_k < \rho \tag{72}$$

So, we have:

$$\frac{\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)^T\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)}{u^2_{\max(l)}} < \overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v \leftrightarrow -\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v + \rho$$
$$\frac{\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)^T\left(\overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k)\right)}{u^2_{\max(l)}} \tag{73}$$

Therefore, by taking Schur complement on the above inequality, we achieve the following LMI that imposes a saturation limit on the control input.

$$\begin{bmatrix} -\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v & * \\ \overline{K}_{hv(l)}(z_k) - \overline{W}_{hv(l)}(z_k) & \frac{u^2_{\max(l)}}{\rho} \end{bmatrix} < 0 \tag{74}$$

■

Proof. Let us consider the following proposed polytopic PD non quadratic Lyapunov function given with the following expression:

$$V\left(\overline{x}_k, z_k\right) = \overline{x}_k^T\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v\overline{x}_k \tag{75}$$

With: $\overline{P}_h(z_k) = \overline{P}_h(z_k)^T > 0, \overline{P}_h(z_k) = diag\left[P_{h1} \quad P_{h2}\right], \overline{E}_v = diag\left[E_v \quad I\right]$.

Another PD nonquadratic Lyapunov function is rewritten by:

$$V\left(\overline{x}_k, z_k\right) = x_k^T E_v^T P_{h1}^{-1} E_v x_k + x_F^T(k) P_{h2}^{-1} x_F(k) \tag{76}$$

Let us define the Lyapunov surface as:

$$\begin{cases} \bar{\mathcal{L}}_v = \{\overline{x}_k \in \mathfrak{R}^{2n_x} : V\left(\overline{x}_k, z_k\right) = \overline{x}_k^T \overline{E}_v^T \overline{P}_h\left(z_k\right)^{-1} \overline{E}_v \overline{x}_k \leq \rho, \forall k > 0\} \\ \bar{\mathcal{L}}_{v0} = \left\{\overline{x}_k \in \mathfrak{R}^{2n_x} : V\left(\overline{x}_k, z_k\right) = \overline{x}_k^T \overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v \overline{x}_k \leq 1, \forall \overline{x}(0) = 0\right\} \end{cases} \tag{77}$$

A simple estimation of $\bar{\mathcal{L}}_{v0}$ and $\bar{\mathcal{L}}_v$ is the intersection of the ellipsoids:

$$\begin{cases} \overline{\mathcal{E}}\left(\overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v, 1\right) = \bigcap_{i=1}^{r} \bigcap_{\hbar=1}^{r_e} \overline{\mathcal{E}}\left(\overline{E}_\hbar^T \overline{P}_i^{-1} \overline{E}_\hbar, 1\right) \subseteq \bar{\mathcal{L}}_{v0} \\ \overline{\mathcal{E}}\left(\overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v, \rho\right) = \bigcap_{i=1}^{r} \bigcap_{\hbar=1}^{r_e} \overline{\mathcal{E}}\left(\overline{E}_\hbar^T \overline{P}_i^{-1} \overline{E}_\hbar, \rho\right) \subseteq \bar{\mathcal{L}}_v \end{cases} \tag{78}$$

The theorem below provides LMI conditions to design the DOFC together with the control problem were: $\bar{\mathcal{L}}_v \subseteq \bar{\mathcal{D}}_{\overline{x}_k} \bigcap \bar{\mathcal{D}}_{u_k}$. ■

Theorem 3. For the given uncertain NLPV descriptor system (1), with a parameter $z_k \in \mathfrak{D}$, under input saturation (8), and the proposed controller (59), whose validity domain is defined in $\bar{\mathcal{D}}_{\overline{x}_k}$, is locally asymptotically stable if there exist symmetric matrices: $P_{i1} = P_{i1}^T > 0, P_{i2} = P_{i2}^T > 0$, where $\{P_{i1}, P_{i2}\} \in \mathfrak{R}^{n_x \times n_x}$; $X_{i1} = X_{i1}^T > 0, X_{i2} = X_{i2}^T > 0$ where $\{X_{i1}, X_{i2}\} \in \mathfrak{R}^{n_x \times n_x}$, a controller gain matrices: $W_{ij}^1 \in \mathfrak{R}^{n_u \times n_x}$; $W_{ij}^2 \in \mathfrak{R}^{n_u \times n_x}$, $L_{ij} \in \mathfrak{R}^{n_x \times n_x}$, $R_{ij} \in \mathfrak{R}^{n_x \times n_x}$, $N_{ij} \in \mathfrak{R}^{n_u \times n_x}$, and $M_{ij} \in \mathfrak{R}^{n_u \times n_x}$, for any positive diagonal gain matrix S_{ij}, a positive scalars $\partial_1, \partial_2, \partial_3, \partial_5, \gamma = \sqrt{\overline{\gamma}}; \gamma^2 = \overline{\gamma}$, δ and ρ, such that condition (15) as satisfied, the following inequalities hold:

$$\begin{cases} min\ \gamma \\ \begin{bmatrix} E_\hbar^T X_{i1} E_\hbar & 0 \\ 0 & X_{i2} \end{bmatrix} > 0 \end{cases} \tag{79}$$

$$\rho + \overline{\gamma}\delta < 1 \tag{80}$$

$$\mathfrak{I}_{ij}^\hbar = \begin{bmatrix} \emptyset_{ij}^\hbar & (*) \\ \Gamma_{ij} & \varphi \end{bmatrix} < 0 \tag{81}$$

$$\emptyset_{ij}^\hbar = \begin{bmatrix} -\mathcal{H}\left(E_\hbar\right) + P_{i1} & 0 & * & * & * & * & * \\ 0 & P_{i2} - 2I & * & * & * & * & * \\ A_i + B_i M_{ij} C & B_i N_{ij} & \begin{matrix} -P_{i1} + \\ \left(\partial_1 H_{ai} H_{ai}^T + (\partial_1 + \partial_2 + \partial_5) H_{bi} H_{bi}^T + \partial_3 H_{e\hbar} H_{e\hbar}^T\right) \end{matrix} & * & * & * & * \\ R_{ij} C & L_{ij} & 0 & -P_{i2} & * & * & * \\ W_{ij}^1 & W_{ij}^2 & -B_i^T S_i^T & 0 & -2S_{ij} & * & * \\ 0 & 0 & B_\omega^T & 0 & 0 & -\overline{\gamma} I & * \\ C & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} \tag{82a}$$

$$\Gamma_{ij} = \begin{bmatrix} N_{ai} & 0 & 0 & 0 & 0 & 0 & 0 \\ N_{bi} M_{ij} C & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_{bi} N_{ij} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{e\hbar} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_{bi} S_{ij} & 0 & 0 \end{bmatrix} \tag{82b}$$

$$\varphi = \begin{bmatrix} -\partial_1 I & * & * & * & * \\ 0 & -\partial_1 I & * & * & * \\ 0 & 0 & -\partial_2 I & * & * \\ 0 & 0 & 0 & -\partial_3 I & * \\ 0 & 0 & 0 & 0 & -\partial_5 I \end{bmatrix} \tag{82c}$$

$$\begin{bmatrix} -E_{\hbar}^T X_{i1} E_{\hbar} & * & * \\ 0 & -X_{i2} & * \\ \mathcal{N}_m^T & 0 & -\frac{1}{\rho} \end{bmatrix} < 0 \tag{83}$$

$$\begin{bmatrix} -E_{\hbar}^T X_{i1} E_{\hbar} & * & * \\ 0 & -X_{i2} & * \\ M_{ij}C - W_{ij}^1 & N_{ij} - W_{ij}^2 & -\frac{\left(u_{(l)}^{max}\right)^2}{\rho} \end{bmatrix} < 0 \tag{84}$$

Then, the modified control law (59) solves the control problem defined for the closed-loop NLPV descriptor system (62). With the gain matrices: A_{ij}^F, B_{ij}^F, C_{ij}^F, and D_{ij}^F were obtained with the bijective transformations as follows: $L_{ij} = A_{ij}^F P_{i2}^{-1}$; $R_{ij} = B_{ij}^F P_{i1}^{-1}$; $N_{ij} = C_{ij}^F P_{i2}^{-1}$; and $M_{ij} = D_{ij}^F P_{i1}^{-1}$, where: $A_{ij}^F = L_{ij} P_{i2}$; $B_{ij}^F = R_{ij} P_{i1}$; $C_{ij}^F(z_k) = N_{ij} P_{i2}$; $D_{ij}^F = M_{ij} P_{i1}$; $X_{i1} = P_{i1}^{-1}$; $X_{i2} = P_{i2}^{-1}$.

Proof. Considering the augmented PD nonquadratic Lyapunov function presented in (75), a saturation control, we end up with:

$$\delta V\left(\overline{x}_k\right) + \overline{x}_k^T \overline{C}^T \overline{C} \overline{x}_k - \gamma^2 \omega_k^T \omega_k - 2\psi^T(u_k) S_{hv}(z_k)^{-1} \left[\psi(u_k) - \overline{W}_{hv}(z_k) \overline{x}_k\right] < 0 \tag{85}$$

From Equation (60), we have:

$$-\overline{E}\overline{x}(k+1) + \overline{A}\overline{x}(k) + \overline{B}\psi(u_k) + \overline{B}_\omega \omega_k = 0 \tag{86}$$

Then inequality (85) can be rewritten as a form:

$$\begin{bmatrix} \overline{x}_k \\ \overline{x}_{k+} \\ \psi(u_k) \\ \omega_k \end{bmatrix}^T \begin{bmatrix} -\overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v + \overline{C}^T \overline{C} & * & * & * \\ 0 & \overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v & * & * \\ S_{hv}(z_k)^{-1} \overline{W}_{hv}(z_k) & 0 & -2S_{hv}(z_k)^{-1} & * \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \overline{x}_k \\ \overline{x}_{k+} \\ \psi(u_k) \\ \omega_k \end{bmatrix} < 0 \tag{87}$$

Then expression (86) can rewritten as:

$$\begin{bmatrix} \overline{\mathrm{A}} & -\overline{\mathrm{E}} & \overline{\mathrm{B}} & \overline{\mathrm{B}}_\omega \end{bmatrix} \begin{bmatrix} \overline{x}_k \\ \overline{x}_{k+} \\ \psi(u_k) \\ \omega_k \end{bmatrix} = 0 \tag{88}$$

Through Finsler's lemma, inequality (87), and equality (88), with $\overline{\mathcal{M}} = \begin{bmatrix} \overline{M} & \overline{N} & 0 & 0 \end{bmatrix}^T$, result in:

$$\begin{bmatrix} -\overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v + \overline{C}^T \overline{C} & * & * & * \\ 0 & \overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v & * & * \\ S_{hv}(z_k)^{-1} \overline{W}_{hv}(z_k) & 0 & -2S_{hv}(z_k)^{-1} & * \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \overline{M} \\ \overline{N} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \overline{\mathrm{A}} & -\overline{\mathrm{E}} & \overline{\mathrm{B}} & \overline{\mathrm{B}}_\omega \end{bmatrix} + (*) < 0 \tag{89}$$

Following the same steps for the development of SOFC, with congruence transformation by $diag\begin{bmatrix} I & I & S_{hv}(z_k)^T & I\end{bmatrix}$, and in order to obtain an LMI problem and a good choice is: $\overline{M} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $\overline{N} = diag\begin{bmatrix} I & I \end{bmatrix}$, we have the previous results.

According to the polytopic PD nonquadartic Lyapunov function, the ellipsoidal DoA $\overline{\mathcal{E}}\left(\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v, 1\right) \subseteq \overline{\mathcal{E}}\left(\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v, \rho\right)$ is always a subset of the polyhedron set $\bar{\mathcal{D}}_{\bar{x}_k}$, if and only if the following inequalities are satisfied:

$$-1 + \bar{\mathcal{N}}_m^T\bar{x}_k + \bar{x}_k^T\bar{\mathcal{N}}_m - \bar{x}_k^T\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v\bar{x}_k < 0 \tag{90a}$$

$$-\frac{1}{\rho} + \bar{\mathcal{N}}_m^T\bar{x}_k + \bar{x}_k^T\bar{\mathcal{N}}_m - \bar{x}_k^T\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v\bar{x}_k < 0 \tag{90b}$$

and

$$\begin{bmatrix} \bar{x}_k \\ 1 \end{bmatrix}^T \begin{bmatrix} -\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v & * \\ \bar{\mathcal{N}}_m^T & -1 \end{bmatrix} \begin{bmatrix} \bar{x}_k \\ 1 \end{bmatrix} < 0 \leftrightarrow \begin{bmatrix} -\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v & * \\ \bar{\mathcal{N}}_m^T & -1 \end{bmatrix} < 0 \tag{91a}$$

$$\begin{bmatrix} \bar{x}_k \\ 1 \end{bmatrix}^T \begin{bmatrix} -\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v & * \\ \bar{\mathcal{N}}_m^T & -\frac{1}{\rho} \end{bmatrix} \begin{bmatrix} \bar{x}_k \\ 1 \end{bmatrix} < 0 \leftrightarrow \begin{bmatrix} -\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v & * \\ \bar{\mathcal{N}}_m^T & -\frac{1}{\rho} \end{bmatrix} < 0 \tag{91b}$$

From Equation (91b), we obtain

$$\begin{bmatrix} -E_{\hbar}^T X_{i1}(z_k) E_{\hbar} & * & * \\ 0 & X_{i1}(z_k) & * \\ \mathcal{N}_m^T & 0 & -\frac{1}{\rho} \end{bmatrix} < 0 \tag{92}$$

Taking into account the ellipsoid: $\bar{\varepsilon}\left(\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v, \rho\right)$, and for all $\bar{x}_k \in \bar{\mathcal{L}}_v\backslash\{0\}$ which implies that $\bar{\mathcal{N}}_m^T\bar{x}_k < 1 \leftrightarrow \mathcal{N}_m^T x_k < 1$, this proves the inclusion $\bar{\mathcal{L}}_v\left(\bar{x}_k\right) \subseteq \bar{\mathcal{D}}_{\bar{x}_k}$, that is proof of Lemma 5.

We multiply $\begin{bmatrix} -\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v & * \\ \left(\overline{K}_{ij}(z_k) - \overline{W}_{ij}(z_k)\right)_{(l)} & -\frac{(u_{\max(l)})^2}{\rho} \end{bmatrix}$ by $diag\begin{bmatrix} \bar{x}_k & 1 \end{bmatrix}$ in the left and by its transpose in the right:

$$\begin{bmatrix} -\bar{x}_k^T\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v\bar{x}_k & * \\ \left(\overline{K}_{ij}(z_k) - \overline{W}_{ij}(z_k)\right)_{(l)}\bar{x}_k & -\frac{(u_{\max(l)})^2}{\rho} \end{bmatrix} < 0 \tag{93}$$

Then: $\bar{x}_k^T\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v\bar{x}_k \leq \rho$ and $\left(\overline{K}_{ij}(z_k) - \overline{W}_{ij}(z_k)\right)_{(l)}\bar{x}_k \leq u_{\max(l)}$.

Applying a Schur complement, inequality (84) holds, from Equation (93), we conclude that: $\bar{\mathcal{L}}_v\left(\bar{x}_k\right) \subseteq \bar{\mathcal{D}}_{u_k}$.

Since $S^{-1} > 0$, by property of the Saturation Lemma, it follows that: $\delta V\left(\bar{x}_k, z_k\right) < 0$, for $\forall\bar{x}_k \subseteq \bar{\mathcal{D}}_{\bar{x}_k} \bigcap \bar{\mathcal{D}}_{u_k}$. This guarantees that the origin of the closed-loop system (60) is asymptotically stable, and $\bar{\mathcal{L}}_v\left(\bar{x}_k\right) \subseteq \bar{\mathcal{D}}_{\bar{x}_k} \bigcap \bar{\mathcal{D}}_{u_k}$ is a contractively invariant set with respect to the NLPV descriptor system (1). This ends the proof. ∎

4.3 | Optimization of the domain of attraction

The preceding results allow the conception of a robust DOFC for NLPV constrained descriptor system (1). An optimization problem design leads to a closed-loop system with a small DoA. In order to maximize the size of the estimate of the DoA, the reference set method proposed in Reference 25, which will be included. The domain of initial condition $\overline{\mathcal{E}}\left(\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v, 1\right)$ is included in the DoA $\overline{\mathcal{E}}\left(\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v, \rho\right)$. This means that the latter can be maximized by optimizing the set of initial condition $\overline{\mathcal{E}}\left(\overline{E}_v^T\overline{P}_h(z_k)^{-1}\overline{E}_v, 1\right)$. Very often, the initial condition $x_F(0) = 0$ of the output feedback controller is set to 0. This information can be utilized in choosing the shape reference set to reduce the conservatism of the estimation for DOFC. In order to obtain a sufficient large estimated DoA, two typical types of $\mathcal{X}_{\mathcal{R}}$ are considered. Consider an ellipsoid $\mathcal{X}_{\mathcal{R}} = \left\{ \overline{x}_k \in \mathfrak{R}^{2n_x} : \overline{x}_k^T \mathcal{R} \overline{x}_k \le 1 \right\}$, where $\mathcal{X}_{\mathcal{R}} \subseteq \mathfrak{R}^{2n_x}$, is a prescribed bounded convex ellipsoid set containing the origin, and $\mathcal{R} > 0$ with compatible dimension.

$$\text{We note}: \overline{\mathcal{E}}\left(\overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v, \rho\right) = \left\{ \overline{x}_k \in \mathfrak{R}^{2n_x} : V\left(\overline{x}_k, z_k\right) = \overline{x}_k^T \overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v \overline{x}_k < \rho \right\} \tag{94}$$

$$\text{And an ellipsoid}: \mathcal{E}\left(E_v^T P_h^{-1} E_v, \rho\right) = \left\{ x_k \in \mathfrak{R}^{n_x} : V\left(x_k, z_k\right) = x_k^T E_v^T P_{h1}^{-1} E_v x_k < \rho \right\} \tag{95}$$

Lemma 7. $\overline{x}_k \in \overline{\mathcal{E}}\left(\overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v, \rho\right)$ if and only if $x_k \in \mathcal{E}\left(E_v^T P_{h1}^{-1} E_v, \rho\right)$.

The transformation exploits the fact that the condition $\overline{P}_h(z_k)^{-1} > 0 \leftrightarrow E_v^T P_{h1}^{-1}(z_k) E_v > 0$ if E_v is invertible. Then:

$$\mathcal{E}\left(E_v^T P_{i1}^{-1}(z_k) E_v, \rho\right) = \mathcal{E}\left(P_{i1}^{-1}(z_k), \rho\right) \tag{96}$$

Proof. Note that:

$$\overline{x}_k \in \overline{\mathcal{E}}\left(\overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v, \rho\right) \leftrightarrow \overline{x}_k \in \overline{\mathcal{E}}\left(\overline{E}_v^T \left(\frac{\overline{P}_h(z_k)^{-1}}{\rho}\right) \overline{E}_v, 1\right) \tag{97}$$

Assume that: $\overline{x}_k \in \overline{\mathcal{E}}\left(\overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v, \rho\right)$ then $x_k \in \mathcal{E}\left(E_v^T P_{i1}^{-1}(z_k) E_v, \rho\right)$ by the inequality:

$$|\, \overline{x}_k^T \overline{E}_v^T \overline{P}_h(z_k)^{-1} \overline{E}_v \overline{x}_k \,| = |\, x_k^T E_v^T P_{h1}^{-1}(z_k) E_v x_k | < \rho \tag{98}$$

$$\left| x_k^T E_v^T P_{h1}^{-1}(z_k) E_v x_k \right| = |x_k^T P_{h1}^{-1}(z_k) x_k| < \rho \tag{99}$$

We can determine the largest ellipsoid $\mathcal{E}\left(E_v^T P_{h1}^{-1} E_v, \rho\right)$ from the following optimization problem:

$$\begin{cases} a)\sup \alpha \\ b)\alpha \mathcal{X}_{\mathcal{R}} \subseteq \bigcap_{i=1}^{r} \bigcap_{\hbar=1}^{r_e} \mathcal{E}\left(E_{\hbar}^T P_{i1}^{-1} E_{\hbar}, 1\right) \end{cases} \tag{100}$$

Constraint (b) is equivalent to: $\alpha^{-2}\mathcal{R} - E_{\hbar}^T P_{i1}^{-1} E_{\hbar} \ge 0$; setting $\alpha^{-2} = \sigma$, using Schur complement,[51] this last inequality is equivalent to:

$$\begin{bmatrix} -\sigma \mathcal{R} & * \\ E_{\hbar} & -P_{i1} \end{bmatrix} < 0 \tag{101}$$

■

Theorem 4. For the given the uncertain NLPV descriptor system (1) under input saturation (8), with a parameter $z_k \in \mathfrak{D}$, and the proposed controller (59), whose validity domain is defined in $\bar{D}_{\overline{x}_k}$, is locally asymptotically stable in a DoA to be maximized if there exist matrices: $P_{i1} = P_{i1}^T > 0$, $P_{i2} = P_{i2}^T > 0$ where $P_{i1} \in \mathfrak{R}^{n_x \times n_x}$and $P_{i2} \in \mathfrak{R}^{n_x \times n_x}$ $X_{i1} = X_{i1}^T > 0$, $X_{i2} = X_{i2}^T > 0$ where $X_{i1} \in \mathfrak{R}^{n_x \times n_x}$ and $X_{i2} \in \mathfrak{R}^{n_x \times n_x}$; a gain matrices $W_{ij}^1 \in \mathfrak{R}^{n_u \times n_x}$; $W_{ij}^2 \in \mathfrak{R}^{n_u \times n_x}$; $A_{ij}^F \in \mathfrak{R}^{n_x \times n_x}$; $B_{ij}^F \in \mathfrak{R}^{n_x \times n_x}$, $C_{ij}^F \in \mathfrak{R}^{n_u \times n_x}$, and $D_{ij}^F \in \mathfrak{R}^{n_u \times n_x}$ for any positive diagonal varying gain matrix $S_{ij} \in \mathfrak{R}^{n_u \times n_u}$, a positive scalars ∂_1, ∂_2, ∂_3, ∂_5, λ, ρ, γ, and σ such that the following LMIs optimization problem are feasible:

$$s.t. \begin{cases} \min\limits_{R, P_{h1} > 0} \sigma \\ LMI\{(79) \sim (84) \text{ and } (101)\} \end{cases} \tag{102}$$

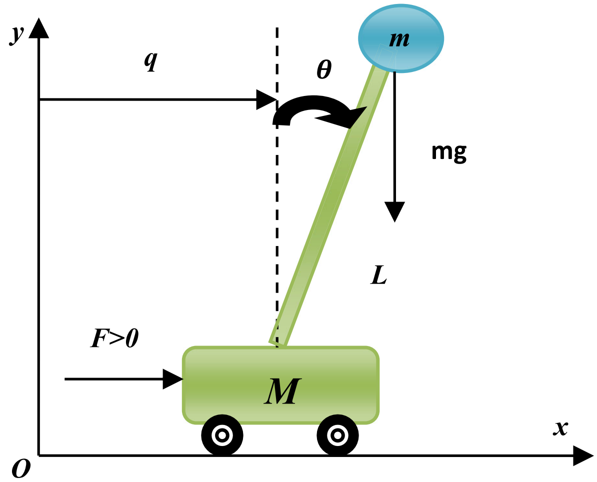

FIGURE 3 Cart-pendulum system [Colour figure can be viewed at wileyonlinelibrary.com]

5 | ILLUSTRATIVE EXAMPLE

Example 1. (28) In order to illustrate the effectiveness of the proposed approach of the design of SOFC, consider the stabilization problem of the unstable nonlinear cart-pendulum system consisting of an inverted pendulum, rotating in a vertical plan around an axis located on a cart moving along the x-axis as shown in Figure 3. Note that the example is used for comparison with other methods proposed in recent published papers.

By using the Euler–Lagrange formulation, the equations of motion of the cart-pendulum system are:

$$\begin{cases} \frac{4}{3}mL^2\ddot{\theta} + mL\ddot{q}\cos(\theta) + b_p\dot{\theta} - mgLsin(\theta) = 0.1\omega(t) \\ mL\ddot{\theta}\cos(\theta) + (M+m)\ddot{q} + b_c\dot{q} - mL\dot{\theta}^2\sin(\theta) = F = satu \end{cases} \tag{103}$$

where q is the cart position, θ is the pendulum angle, and F is the force applied to the cart. By denoting the state variable $x = \begin{bmatrix}\theta & q & \dot{\theta} & \dot{q}\end{bmatrix}^T$ and the saturated control input: $satu = F$, the nonlinearities as follows:

$$z_1 = mL\cos(x_1), z_2 = mgLsinc(x_1) \text{ and } z_3 = mLx_3\sin(x_1) \tag{104}$$

Then,

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \frac{4}{3}mL^2\dot{x}_3 + z_1\dot{x}_4 + b_px_4 - z_2x_1 = 0.1\omega(t) \\ z_1\dot{x}_3 + (M+m)\dot{x}_4 + b_cx_4 - z_3x_3 = F = satu \\ y_k = Cx_k \end{cases} \tag{105}$$

Using Euler's method, system (105) becomes:

$$\begin{cases} x_1(k+1) = x_1(k) + T_ex_2(k) \\ x_2(k+1) = x_2(k) + T_ex_3(k) \\ \frac{4}{3}mL^2x_3(k+1) + z_1x_4(k+1) = \frac{4}{3}mL^2x_3(k) + \left(z_1 - T_eb_p\right)x_4(k) + T_ez_2x_1(k) + 0.1T_e\omega(k) \\ z_1x_3(k+1) + (M+m)x_4(k+1) = (z_1 + z_3T_e)x_3(k) + (M+m-b_cT_e)x_4(k) + T_esatu(\mathrm{k}) \\ y_k = Cx_k \end{cases} \tag{106}$$

Making use of the descriptor polytopic form, the nonlinear dynamic model of the cart-pendulum system can be rewritten as:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{4}{3}mL^2 & z_1 \\ 0 & 0 & z_1 & (M+m) \end{bmatrix} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T_e & 0 & 0 \\ 0 & 1 & T_e & 0 \\ T_e z_2 & 0 & \frac{4}{3}mL^2 & z_1 - T_e b_p \\ 0 & 0 & (z_1 + z_3 T_e) & (M+m-b_c T_e) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ T_e \end{bmatrix} satu(k) + \begin{bmatrix} 0 \\ 0 \\ 0.1T_e \\ 0 \end{bmatrix} \omega(k) \tag{107}$$

Where:

$$E_{h}(z_1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{4}{3}mL^2 & z_1 \\ 0 & 0 & z_1 & (M+m) \end{bmatrix}; A_h(z_1, z_2, z_3) = \begin{bmatrix} 1 & T_e & 0 & 0 \\ 0 & 1 & T_e & 0 \\ T_e z_2 & 0 & \frac{4}{3}mL^2 & z_1 - T_e b_p \\ 0 & 0 & (z_1 + z_3 T_e) & (M+m-b_c T_e) \end{bmatrix};$$

$$B_h = \begin{bmatrix} 0 \\ 0 \\ 0 \\ T_e \end{bmatrix} \text{ and } B_\omega = \begin{bmatrix} 0 \\ 0 \\ 0.1T_e \\ 0 \end{bmatrix}.$$

By using the sector nonlinearity approach,[49] the nonlinear system (105) can be expressed based on the uncertain NLPV descriptor form (1), where the premise variables are chosen as (104).

In order to simplify the model, the terms weighted by: $mL\cos(x_1)$ and $z_{3k} = mLx_{3k}\sin(x_{1k})$ will be considered as uncertainties. Their maximal magnitudes taken on the domain $\mathcal{D}_x$ are, respectively: $z_{1max} = 0.0394$ and $z_{3max} = 0.1968$, then we define:

$$E_1 = E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{4}{3}mL^2 & 0 \\ 0 & 0 & 0 & (M+m) \end{bmatrix}; A_1(z_{2min}) = \begin{bmatrix} 1 & T_e & 0 & 0 \\ 0 & 1 & Te & 0 \\ T_e z_{2min} & 0 & \frac{4}{3}mL^2 & -T_e b_p \\ 0 & 0 & 0 & (M+m-b_c T_e) \end{bmatrix};$$

$$A_2(z_{2max}) = \begin{bmatrix} 1 & T_e & 0 & 0 \\ 0 & 1 & Te & 0 \\ T_e z_{2max} & 0 & \frac{4}{3}mL^2 & -T_e b_p \\ 0 & 0 & 0 & (M+m-b_c T_e) \end{bmatrix}; B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ T_e \end{bmatrix}; B_\omega = \begin{bmatrix} 0 \\ 0 \\ 0.1T_e \\ 0 \end{bmatrix}; \text{ and } C = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Therefore, the polytopic coordinates are defined as:

$$\begin{cases} h_1(z_{2k}) = \frac{z_{2k} - z_{2kmin}}{z_{2kmax} - z_{2kmin}} \\ h_2(z_{2k}) = 1 - h_1(z_{2k}) \end{cases}$$

and

$$H_{a1} = H_{a2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ T_e \end{bmatrix}; N_{a1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ z_{1max} \end{bmatrix}^T; N_{a2} = \begin{bmatrix} 0 \\ 0 \\ z_{1max} + T_e z_{3max} \\ 0 \end{bmatrix}^T; H_{e1} = H_{e2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ T_e \end{bmatrix};$$

$$N_{e1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ z_{1max} \end{bmatrix}^T; N_{e2} = \begin{bmatrix} 0 \\ 0 \\ z_{1max} \\ 0 \end{bmatrix}^T.$$

Now, using physical parameters reported in Table 1, with chosen bounds as: $\mathcal{D}_x = \left\{ |\theta| \leq \frac{41\pi}{180} [rad]; |\dot{\theta}| \leq 5 [rad.s^{-1}] \right\}$, $\mathcal{N}_m = \left\{ N_1 = \begin{bmatrix} \frac{180}{41\pi} & 0 & 0 & 0 \end{bmatrix}, N_2 = \begin{bmatrix} 0 & 0 & \frac{1}{5} & 0 \end{bmatrix} \right\}$, the maximal saturation level defined as: $|u_{\max(l)}| \leq 1$, the initial condition $x(0) = \begin{bmatrix} \frac{30\pi}{180} & 0 & 0 & 0 \end{bmatrix}$, the persistent disturbance signal bounded in amplitude is: $\omega(k) = \sin(20k)$, sampling time is $T_e = 0.8[s]$, and a maximal level of the disturbances given as: $\delta = 0.2$.

TABLE 1 Physical parameters of the cart-Pendulum system

Mass of cart	M	**0.5 (kg)**
Mass of the pendulum	m	0.2 (kg)
Length to the pendulum center of mass	L	0.3 (m)
Length to the pendulum center of mass	b_p	0.01 (kg)
Viscous friction coefficient of the cart	b_c	0.1 (Nsm^{-1})
Gravity	g	9.81 (ms^{-2})

The goal of the first example is to compare the presented results with respect to those given in Reference 26. To show the validity of our approach, first we solve the optimization problem in Theorem 1, by solving the LMIs conditions. All computations are carried out under MATLAB software with the LMI solver of the robust control Toolbox, resulting in feasible solutions. The following matrices are obtained using Theorem 1:

$$P_1 = \begin{bmatrix} 0.5435 & -0.0016 & -0.0751 & 0.0097 \\ -0.0016 & 0.6179 & -0.0003 & 0.0373 \\ -0.0751 & -0.0003 & 0.1481 & 0.0047 \\ 0.0097 & 0.0373 & 0.0047 & 0.3552 \end{bmatrix}; P_2 = \begin{bmatrix} 0.5404 & -0.0013 & -0.0683 & 0.0067 \\ -0.0013 & 0.6179 & -0.0003 & 0.0371 \\ -0.0683 & -0.0003 & 0.1472 & 0.0029 \\ 0.0067 & 0.0371 & 0.0029 & 0.3577 \end{bmatrix}.$$

The obtained feedback gains of SOFC are obtained as follows:

$$F_{11} = F_{21} = \begin{bmatrix} 0.0141 & -0.0195 & -0.0118 & 0.0416 \end{bmatrix}; F_{12} = F_{22} = \begin{bmatrix} 0.0129 & -0.0195 & -0.0085 & 0.0413 \end{bmatrix}.$$

$$H_{11} = H_{21} = \begin{bmatrix} -0.0066 & -0.008 & -0.0318 & 0.0217 \\ -0.0021 & 0.0179 & -0.0008 & 0.0244 \\ 0.0025 & 0.0004 & 0.0075 & -0.0107 \\ -0.0066 & 0.0053 & 0.0134 & -0.0858 \end{bmatrix};$$

$$H_{12} = H_{22} = \begin{bmatrix} -0.0067 & -0.0007 & -0.0228 & 0.0165 \\ -0.0020 & 0.0180 & -0.0005 & 0.0241 \\ 0.0046 & 0.0005 & 0.0117 & -0.0157 \\ -0.0055 & 0.0053 & 0.0094 & -0.0836 \end{bmatrix}.$$

and the scalars: $\varepsilon_1 = 0.8871$; $\varepsilon_2 = 0.6989$; $\varepsilon_3 = 0.8872$; $\gamma = 0.9419$; and $\rho = 0.3790$.

5.1 | Discussion

The obtained simulation results, reported in Figures 4 to 6, show the closed-loop behavior using the proposed controller. From the plotted graphs, the following observations are drawn:

- The SOF saturated control in this work is robust, because it tends toward zero after $t = 7[s]$with respect to the variables disturbances; the control trajectories are depicted in Figure 4.
- The curves of the state responses of x_k are shown in Figures 5 and 6, which indicate that the inverted pendulum system is stable under saturation input and disturbances, and then the desired algorithm is feasible.
- According to the previous results, it can be stipulated that the addition of the variable disturbance to nonlinear system never influences the performance of the closed-loop system.
- Also, it is noteworthy that, despite both the saturation actuators and the noises at the beginning of simulation, the nonlinear controller guarantees an asymptotic stability of the closed-loop system. This demonstrates the effectiveness of the proposed control approach in dealing with complex constrained nonlinear systems in real word applications. Moreover, it is important to highlight that the proposed approach ensures the stability of nonlinear system, that may be

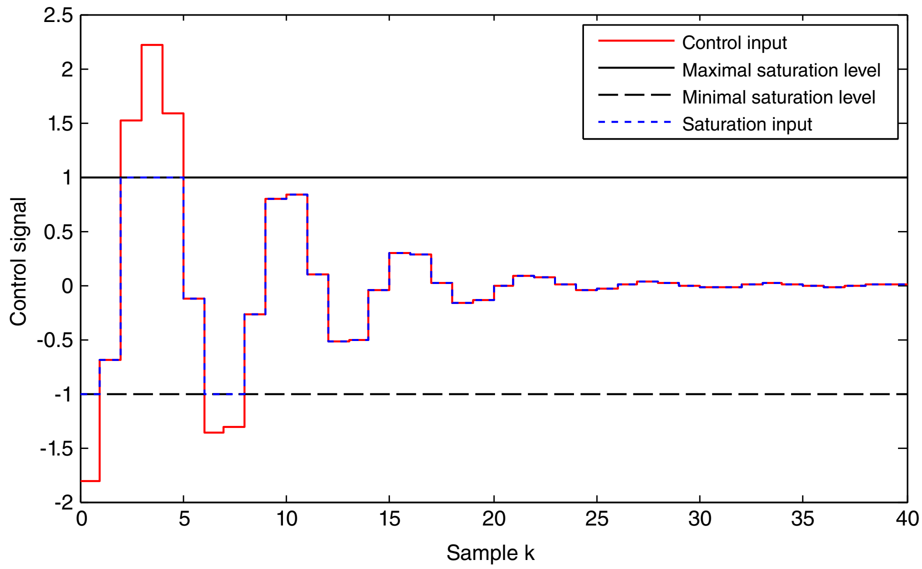

FIGURE 4 Command signals for the cart-pendulum system [Colour figure can be viewed at wileyonlinelibrary.com]

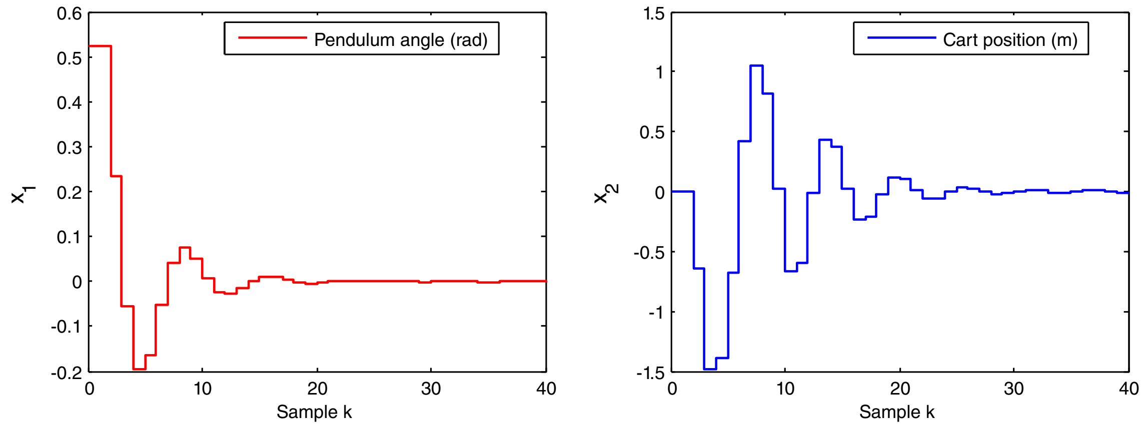

FIGURE 5 Position responses of the cart-pendulum system [Colour figure can be viewed at wileyonlinelibrary.com]

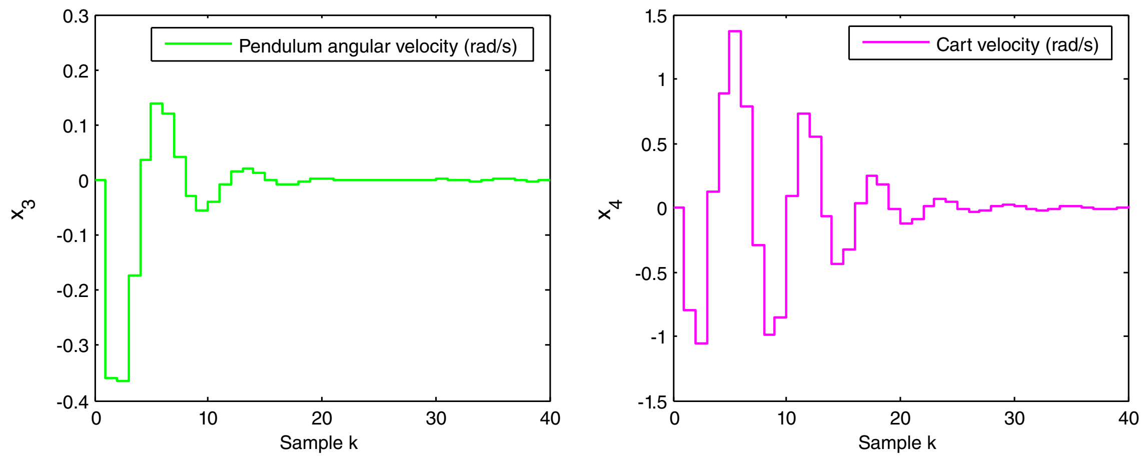

FIGURE 6 Velocity responses of the cart-pendulum system [Colour figure can be viewed at wileyonlinelibrary.com]

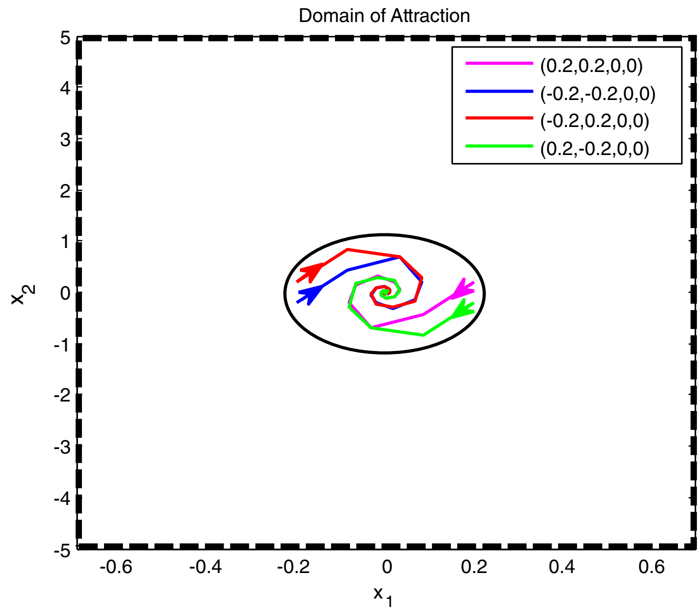

FIGURE 7 Projection of DoA onto $x_1 - x_2$ plan [Colour figure can be viewed at wileyonlinelibrary.com]

destabilized by the control saturation, when the last, is not taken into account in the control synthesis. It can clearly be concluded that our developed method is correct and robust, and constitutes a very good property for inverted pendulum with a good disturbance reject.

Next, we solve the optimization problem in Theorem 2 with: $\mathcal{R} = I$. The following figure depicts the projection of the DoA onto the $x_1 - x_2$ plan ($x_3 = 0, x_4 = 0$). The closed-loop trajectories for various initial condition are depicted inside the validity domain (dashed line).

This figure shows the guaranteed region of attraction $\mathcal{E}\left(E_{\hbar}^T P_i^{-1} E_{\hbar}, 1\right)$ of the closed-loop system which is maximized inside $\mathcal{D}_x$, by means of simulation it can be checked that the corresponding SOFC provides stable closed-loop behavior for $\forall x(0) \in \mathcal{D}_x$. The size of $\mathcal{E}\left(E_{\hbar}^T P_i^{-1} E_{\hbar}, 1\right)$ is maximized because the intersection size of all ellipsoids $\mathcal{E}\left(E_v^T P_h^{-1} E_v, 1\right)$ is maximized by solving LMIs (58).

We observe that the closed-loop system is stabilized and the disturbance is well attenuated. By means of simulation, it can be also checked that the obtained controller provides stable behaviors for all closed-loop trajectories initialized in $\mathcal{D}_x$ and these trajectories remain inside this validity domain (Figure 7).

Note that the DoA involving with P^* (P optimal) is obtained by solving LMIS (58). It can be easily seen that the system states initialized in this maximal DoA will convergence to the origin and their trajectories never escape this domain, the minimal value of $\eta = 2.0455$, was obtained for:

$$P_1^* = \begin{bmatrix} 2.1937 & -0.0035 & -0.3381 & 0.0296 \\ -0.0035 & 2.1951 & -0.0004 & 0.0149 \\ -0.3381 & -0.0004 & 0.2026 & 0.0133 \\ 0.0296 & 0.0149 & 0.0133 & 1.5867 \end{bmatrix}; P_2^* = \begin{bmatrix} 2.1783 & -0.0045 & -0.3054 & 0.0113 \\ -0.0045 & 2.4249 & -0.0016 & 0.1049 \\ -0.3054 & -0.0016 & 0.1973 & 0.0056 \\ 0.0113 & 0.1049 & 0.0056 & 1.6379 \end{bmatrix}.$$

It can also be observed in Figures 4 to 6 that the simulated closed-loop trajectories with initial states taken on the boundary of $\mathcal{D}_x$ and converge to the origin.

Example 2. (48) For DOFC, we consider a nonlinear descriptor system, as shown in Figure 8, where the model describes a disk rolling on a surface without slipping. The disk is connected to a fixed wall via a nonlinear spring and a linear damper, where $x_1(t)$is represents the position of the disk center, $x_2(t)$ is the speed of translation of the disk, $x_3(t)$ is the angular speed of the disk, and $x_4(t)$ represents the contact force between the disk and the surface.

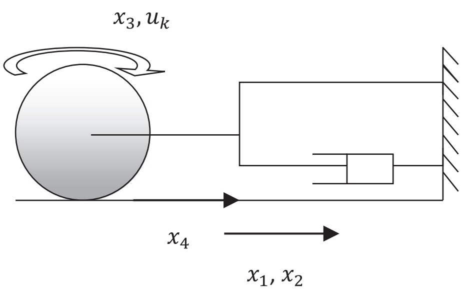

FIGURE 8 A roller disk

This nonlinear model can be approximated by a multi-model, composed of local models. Hence, the multi-model rolling disk representation can be given by the following state space:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = z_1 x_1(t) - \frac{b}{m} x_2(t) + \frac{1}{m} x_4(t) + satu(t) + \omega(t) \\ 0 = x_2(t) - r x_3(t) \\ 0 = -\frac{b}{m} x_2(t) - z_2 x_3(t) + \left(\frac{r^2}{J} + \frac{1}{m}\right) x_4(t) - \frac{r}{J} satu(t) \\ y_1(t) = x_1(t) + x_3(t) \\ y_2(t) = x_3(t) \\ y_3(t) = x_2(t) + x_4(t) \\ y_4(t) = x_3(t) \end{cases} \tag{108}$$

By denoting the state variable vector: $x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$, let us define the nonlinearities as follows:

$$z_1 = \frac{K}{m\left(1 + x_1^2\right)}, z_2 = \frac{K}{m\left(1 + x_3^2\right)} \tag{109}$$

Using Euler's method, the equations of roller disk system are defined by:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1 \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T_e & 0 & 0 \\ T_e z_1 & 1 - \left(\frac{bT_e}{m}\right) & 0 & \frac{T_e}{m} \\ 0 & 1 & -r & 0 \\ 0 & -\frac{b}{m} & -z_2 & \left(\frac{r^2}{J} + \frac{1}{m}\right) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} 0 \\ T_e \\ 0 \\ -\frac{r}{J} \end{bmatrix} satu(k) + \begin{bmatrix} 0 \\ T_e \\ 0 \\ 0 \end{bmatrix} \omega(k) \tag{110}$$

$$\text{and} \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \\ y_4(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} \tag{111}$$

Using the sector nonlinearity approach,[49] the nonlinear system (110) and (111) can be expressed base on the uncertain NLPV descriptor form (1), where the polytopic coordinates are chosen as (109).

In order to simplify the model, the term weighted by: $z_{1k} = \frac{K}{m\left(1+x_1^2\right)}$, will be considered as uncertainties. Their maximal magnitudes taken on the domain $\bar{D}_{\bar{x}_k}$ is: $z_{1max} = 1$, then:

TABLE 2 Physical parameter of roller disk

A spring coefficient of roller disk	K	100 $(N.\ m^{-1})$
A damping coefficient of roller disk	b	30
Mass of disk	m	40 (kg)
The disk radius	r	0.4 (m)

We define: $E_1 = E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; A_1(z_{2min}) = \begin{bmatrix} 1 & T_e & 0 & 0 \\ 0 & 1-\left(\frac{bT_e}{m}\right) & 0 & \frac{T_e}{m} \\ 0 & 1 & -r & 0 \\ 0 & -\frac{b}{m} & -z_{2min} & \left(\frac{r^2}{J}+\frac{1}{m}\right) \end{bmatrix};$

$$A_2(z_{2max}) = \begin{bmatrix} 1 & T_e & 0 & 0 \\ 0 & 1-\left(\frac{bT_e}{m}\right) & 0 & \frac{T_e}{m} \\ 0 & 1 & -r & 0 \\ 0 & -\frac{b}{m} & -z_{2max} & \left(\frac{r^2}{J}+\frac{1}{m}\right) \end{bmatrix}; B_1 = B_2 = \begin{bmatrix} 0 \\ T_e \\ 0 \\ -\frac{r}{J} \end{bmatrix}; B_\omega = \begin{bmatrix} 0 \\ T_e \\ 0 \\ 0 \end{bmatrix}; \text{and } C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Therefore, the membership functions and the known constant matrices are defined as:

$$\begin{cases} h_1(z_2) = \frac{z_2 - z_{2min}}{z_{2max} - z_{2min}} \\ h_2(z_2) = 1 - h_1(z_2) \end{cases}; H_{a1} = H_{a2} = \begin{bmatrix} 0 \\ T_e \\ 0 \\ -\frac{r}{J} \end{bmatrix}; N_{a1} = N_{a2} = \begin{bmatrix} -T_e z_{1max} \\ 0 \\ 0 \\ 0 \end{bmatrix}^T.$$

By using physical parameters given in Table 2, with chosen bounds as: $\bar{D}_{\bar{x}_k} = \left\{ |x_{1k}| \le \pi \ [rad]; |x_{2k}| \le \frac{\pi}{2} \ [rad.s^{-1}] \right\}$, the maximal saturation level defined as: $|u_{\max(l)}| \le 1$, the sampling time is $T_e = 0.25\ [s]$.

The main advantage of the proposed approach is to synthesize the control law by considering the saturation limits. Then the goal of this example aims at designing a robust $\mathfrak{L}_2$ controller that guarantee the attenuation of the external disturbances ω_k affecting the system output and state vector. The DOFC gain matrices, ensuring a minimized $\mathfrak{L}_2$ attenuation level value γ, are obtained by solving an optimization problem under LMI constraints using the Matlab LMI Toolbox through Theorem 3. This solution could be improved since the nonquadratic LMI conditions, are reputed of less conservatism.[25]

For simulation realization, the system is energy bounded disturbance signal ω_k and of the following form:

$$\omega_k = \begin{cases} -15 * \cos\left(k - \frac{\pi}{3}\right); \text{for } k \in [30 \quad 36] \\ 0 \quad ; \text{elsewhere} \end{cases} \tag{112}$$

Also, the corresponding energy-bounded disturbance of ω_k is selected as:$\delta = 0.2$ happening at $t = 7.5[s]$ with duration 1.25 $[s]$. This provides the DOFC gain matrices given by:

$$A_{11}^F = A_{21}^F = \begin{bmatrix} 0.2132 & -0.5270 & -0.0491 & 0.0528 \\ -0.3321 & 0.8295 & 0.0755 & -0.0997 \\ -0.0948 & 0.2425 & 0.0221 & -0.0381 \\ -0.3283 & 0.8229 & 0.0747 & -0.1022 \end{bmatrix};$$

$$A_{12}^F = A_{22}^F = \begin{bmatrix} 0.2087 & -0.5678 & -0.0501 & 0.0825 \\ -0.3264 & 0.8866 & 0.0771 & -0.1410 \\ -0.0930 & 0.2591 & 0.0226 & -0.0502 \\ -0.3227 & 0.8790 & 0.0763 & -0.1428 \end{bmatrix};$$

$$B_{11}^F = B_{21}^F = \begin{bmatrix} 0.0987 & -0.0911 & -0.8000 & 0.0054 \\ 0.0321 & -0.0238 & 0.0090 & -0.1050 \\ -0.0765 & -0.8030 & 0.0006 & -0.1209 \\ 0.0321 & 0.0238 & -0.0090 & 0.1050 \end{bmatrix};$$

$$B_{12}^F = B_{22}^F = \begin{bmatrix} 0.3206 & 0.0106 & 0.0804 & 0.0021 \\ 0.0471 & 0.0021 & -0.0139 & 0.0720 \\ 0.0985 & 0.1026 & 0.0432 & 0.0406 \\ -0.0471 & -0.0021 & 0.0139 & -0.0720 \end{bmatrix};$$

$$C_{11}^F = C_{12}^F = C_{21}^F = C_{22}^F = [0.0115 \quad -0.0040 \quad -0.0112 \quad -0.0008];$$

$$D_{11}^F = D_{21}^F = [0.2400 \quad -5.5365 \quad 0.0431 \quad 5.2464]; D_{12}^F = D_{22}^F = [-0.3520 \quad 4.1104 \quad 0.0995 \quad -3.6694].$$

$$P_1 = \begin{bmatrix} 0.0715 & -0.0028 & 0.0015 & -0.0004 \\ -0.0028 & 0.0951 & 0.0034 & -0.0221 \\ 0.0015 & 0.0034 & 0.0494 & -0.0028 \\ -0.0004 & -0.0221 & -0.0028 & 0.0590 \end{bmatrix}; P_2 = \begin{bmatrix} -1.4574 & 0.0010 & 0.3190 & 0.0212 \\ 0.0010 & -1.4574 & 1.3098 & 0.0003 \\ 0.3190 & 1.3098 & -1.4574 & 1.0020 \\ 0.0212 & 0.0002 & 1.0020 & -1.4574 \end{bmatrix}$$

$$\partial_1 = 1.0319; \partial_2 = 1.1228; \partial_3 = 0.8935; \partial_5 = 0.0319; \gamma = 0.9801; \text{and } \rho = 0.2672.$$

Applying the controller gain matrices given previously, Figure 9 shows the control input signal and Figure 10 depicts the closed-loop response from an initial state fixed at $x(0) = \begin{bmatrix} 1 & -\frac{\pi}{3} & 0 & 0 \end{bmatrix}^T$, with the controller being designed using the optimization approach. It can be seen that the system is stable even in the presence of control input saturation, where two phases can be distinguished. For the first phase from $t = 0[s]$ to $t = 7.5\ [s]$, the disturbance is not involved in the dynamics of the roller disk system, and all four state variables converge to the origin. The second phase is from $t = 7.5\ [s]$ until the end of the simulation. It can be observed that at $t = 7.5\ [s]$, the considered disturbance ω_k begins to act on the system dynamics for duration of 1.25 $[s]$, perturbing, therefore, the system state variables. However, the proposed robust controller is able to reject the disturbance effect and all states converge again to the origin at the end of the simulation (Figure 11).

Now, by solving an optimization problem under LMI constraints using Theorem 4, chosen $\mathcal{R} = I$.

The set $\mathcal{E}\left(E_v^T P_{h1}^{-1} E_v, 1\right)$ is always contained in $\overline{\mathcal{E}}\left(\overline{E}_v^T \overline{P}_h^{-1} \overline{E}_v, 1\right)$, assuming that $\overline{x}(0) \in \overline{\mathcal{E}}\left(\overline{E}_v^T \overline{P}_h^{-1} \overline{E}_v, 1\right)$ if $x(0) \in \mathcal{E}\left(E_v^T P_{h1}^{-1} E_v, 1\right)$. In the absence of disturbances, the set $\mathcal{E}\left(E_v^T P_{h1}^{-1} E_v, 1\right)$ is positively invariant and so $\overline{x}(k) \in$

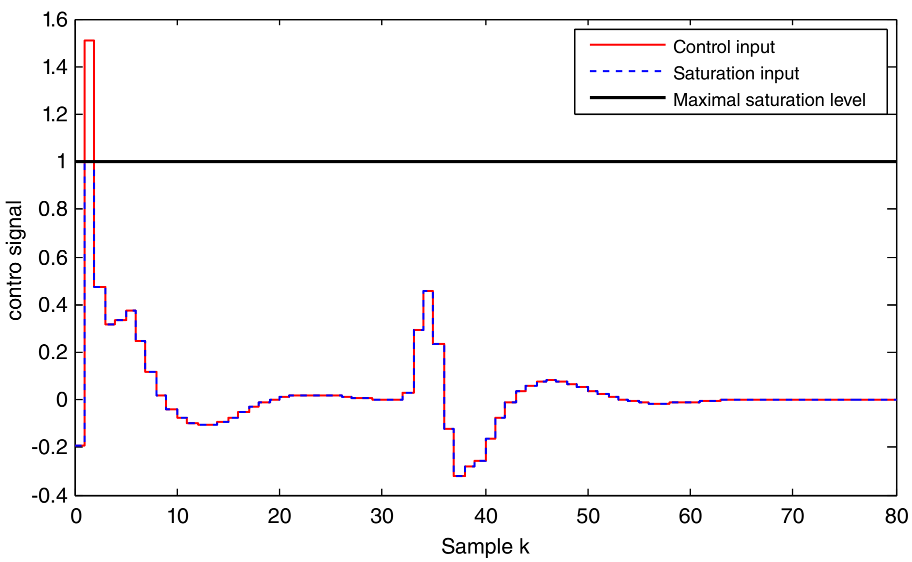

FIGURE 9 Control signal [Colour figure can be viewed at wileyonlinelibrary.com]

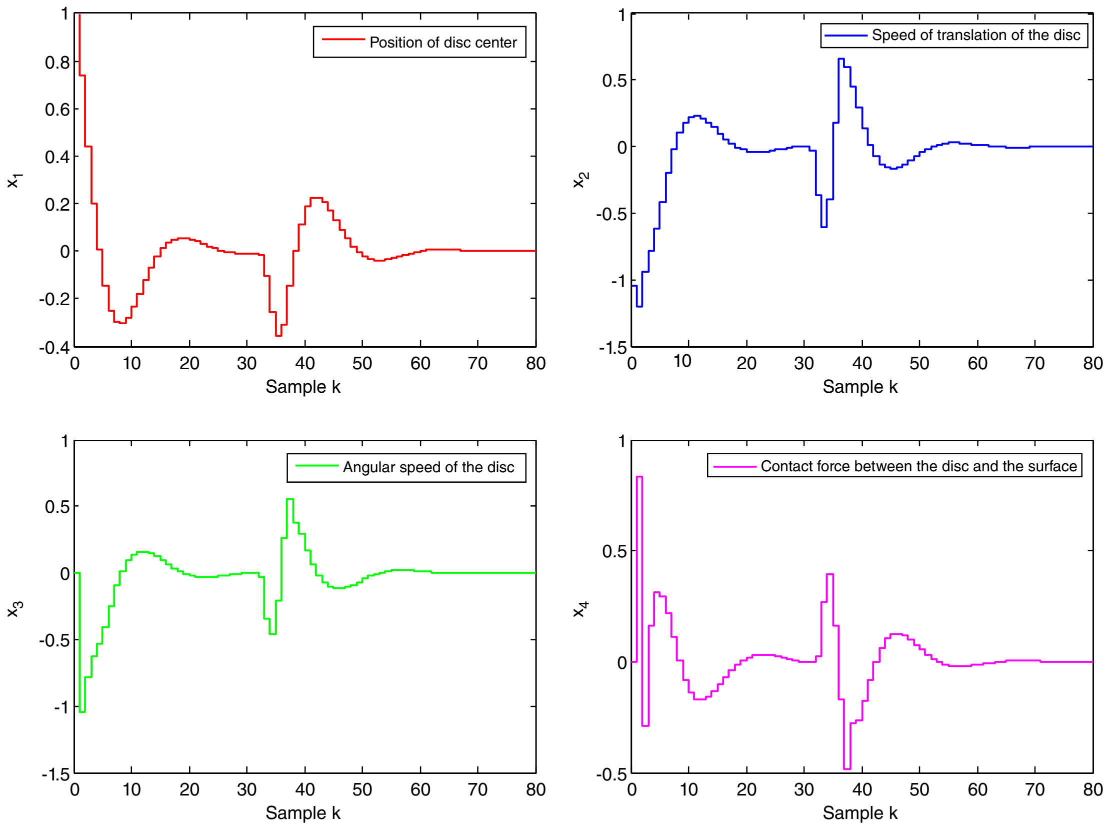

FIGURE 10 System's state signals [Colour figure can be viewed at wileyonlinelibrary.com]

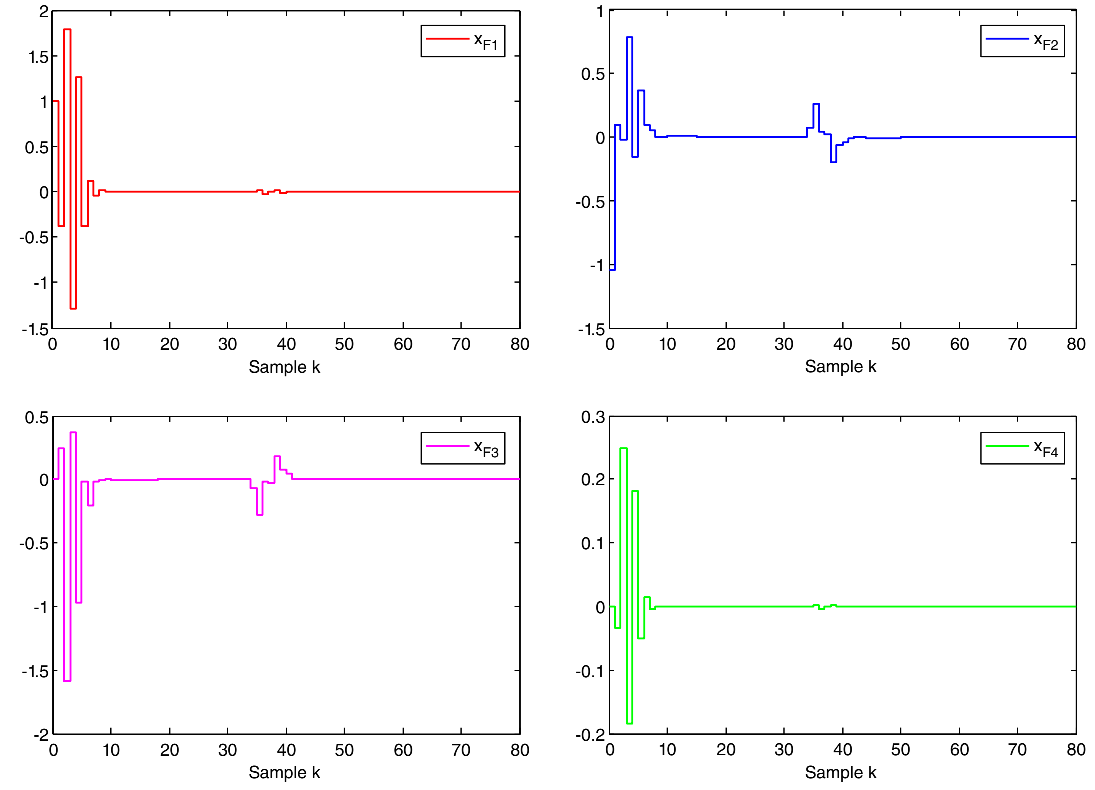

FIGURE 11 Controller's state signals [Colour figure can be viewed at wileyonlinelibrary.com]

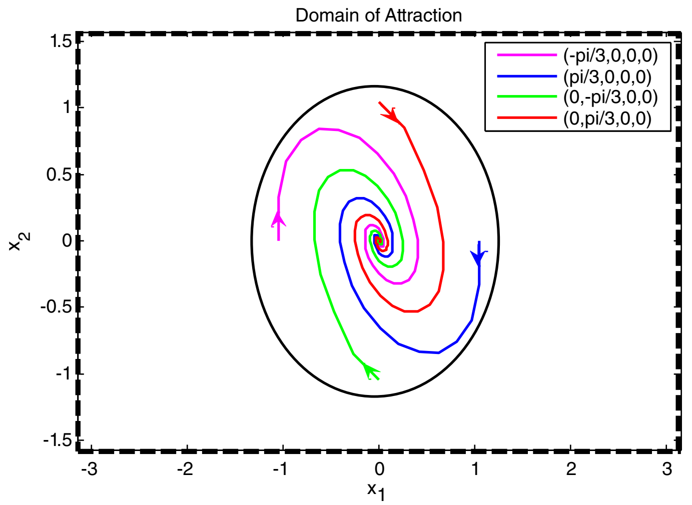

FIGURE 12 Curves of phase diagrams $x_1 - x_2$ plan [Colour figure can be viewed at wileyonlinelibrary.com]

$\overline{\mathcal{E}}\left(\overline{E}_v^T\overline{P}_h^{-1}\overline{E}_v, 1\right)$, for all k ≥ 0, therefore, x_k will never escape the projection of $\overline{\mathcal{E}}\left(\overline{E}_v^T\overline{P}_h^{-1}\overline{E}_v, 1\right)$ onto the plane defined by $x_F = 0$.

Observe that, simulation of several trajectories initialized on the boundary of the corresponding set $\overline{\mathcal{E}}\left(\overline{E}_v^T\overline{P}_h^{-1}\overline{E}_v, 1\right)$, has been depicted with various initial condition in Figure 12. Thus, the obtained results confirm that these set are positively invariant with respect to the closed-loop dynamics and contained in the validity domain (dashed line) of the equilibrium state $x_k = x_F = 0$, with $\sigma = 0.7975$, with:

$$P_1^* = \begin{bmatrix} 0.9715 & -0.1028 & 0.3315 & -0.0422 \\ -0.1028 & 0.2951 & 0.0034 & -0.3221 \\ 0.3315 & 0.0034 & 0.6494 & -0.9928 \\ -0.0422 & -0.3221 & -0.9928 & 0.2590 \end{bmatrix}; P_2^* = \begin{bmatrix} -3.5580 & 0.0010 & 0.3190 & 0.9212 \\ 0.0010 & -3.5580 & 2.3098 & 0.0067 \\ 0.3190 & 2.3098 & -3.5580 & 1.0220 \\ 0.9212 & 0.0067 & 1.0220 & -3.5580 \end{bmatrix}.$$

6 | CONCLUSION

In this article, the problem of output feedback stabilization of uncertain discrete-time NLPV descriptor systems subject to actuator saturation and external disturbances has been tackled. Both SOFC and DOFC have been developed. $\mathfrak{L}_2$ criterion has been employed to ensure a minimal attenuation level of external disturbances. The estimation of the largest DoA has been formulated and solved as a LMIs convex optimization problem. Relaxed LMI conditions for the defined problem have been easily obtained, and the controller gains are efficiently computed with programming solvers. Finally, two examples have been used to show the performance of the proposed approaches. As future works, we will focus on observer-based control design for fuzzy/NLPV descriptor systems under state, input constraints, and exogenous disturbance. The case of system with unmeasurable premise variables will be treated. A second interesting perspective will be to apply the obtained results to a practical constrained plant in order to maintain stability, acceptable dynamic performance, and steady state of the system despite input system constraints.

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